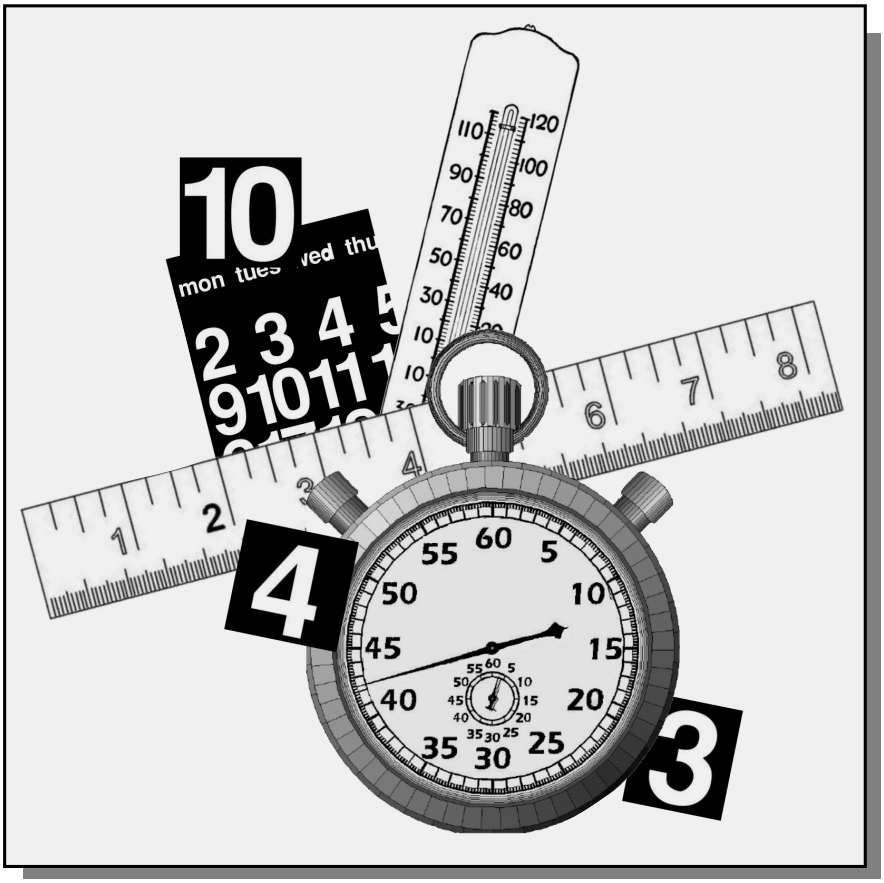
Basic Math Skills



Chapter 3

BMS: 76025 Revision: 030208

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

PROFESSIONAL CREDITS

Basic Math Skills

Chapter 3

Publisher and Managing Editor
Ronald E. Johnson, Ph.D.

Writers
Kermit Northcutt
David Browder

Technical Editor
David Browder

Content Editor / QA
Nancy Johnson

Production Manager
H. Michael Young

Page Composition
Debbie L. Dial
Kathy L. Dial
Pamela K. Hurst
H. Michael Young

Graphic Artists
Kathy L. Dial
Pamela K. Hurst
Kristin Odom
Lydia Seals
Don Ward
Dan White
H. Michael Young

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

CHAPTER 3
TABLE OF CONTENTS

PAGE

SECTION 1

1. GRAPHS 1-4
Cartesian Coordinate System

2. SIGNED NUMBERS 5-8
Definitions, Addition and Subtraction with Like and Unlike Signs

3. SIGNED NUMBERS 9-11
Multiplication and Division with Like and Unlike Signs

4. GROUPING 13-16
Rules, Operation Symbols

5. ASSOCIATIVE AND DISTRIBUTIVE PROPERTIES 17-19
Unknowns, Variables

SECTION 2

6. EXPONENTIALLY SPEAKING 21-23
Properties, Form

7. PEMDAS 25-28
Order of Operations

8. SQUARE POWER 29-32
Area of Squares and Rectangles

PAGE

9. SQUARE ROOTS 33-36
Principal Square Root, Perfect Square

10. CUBES 37-39
Relationship to Volume

SECTION 3

11. CUBE ROOTS 41-43
Definition, Extraction

12. ENGLISH WEIGHT 45-47
Conversions

13. METRIC WEIGHT 49-51
Conversions

14. FRACTIONAL MEASURE 53-56
Volume, Length, Weight

15. OF SOME INTEREST 57-59
Simple Interest Formula

MATHEMATICS CHARTS 61-63



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

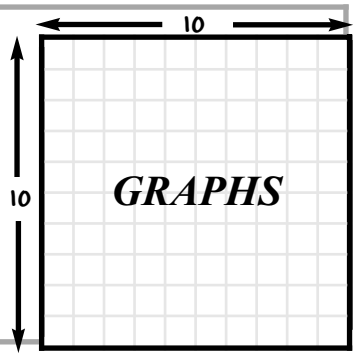
Basic Math Skills



Graphs

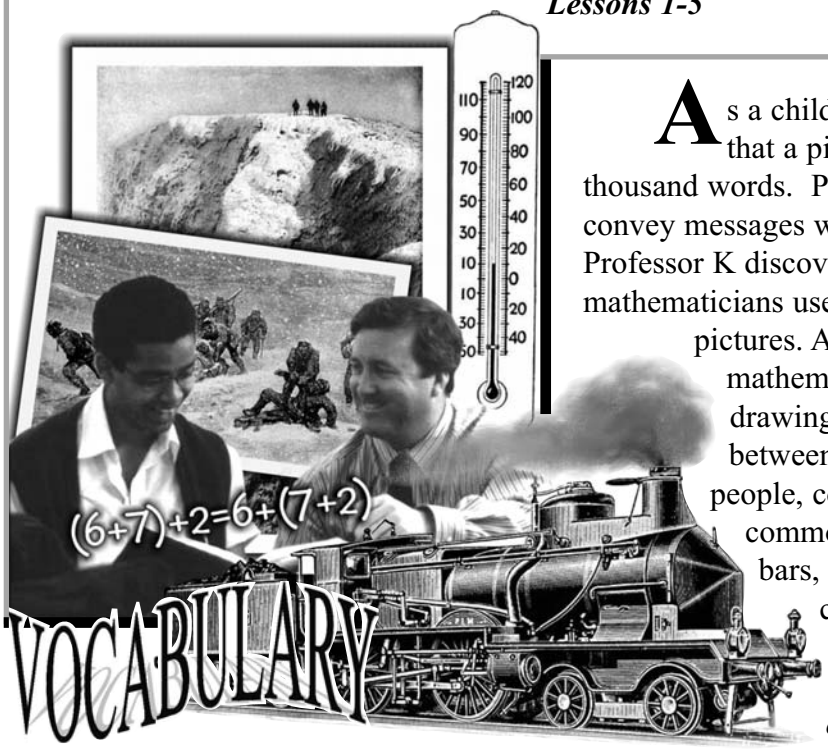
Lesson 1

Chapter 3



Section 1

Lessons 1-5



Graph: a pictorial representation of the comparison of two or more sets of numbers

Stock market: an establishment for the trading of stocks

Commodities: quantities of products, goods or raw materials used for trade or commerce

Tick mark: a mark on a graph used to draw the viewer's attention to that particular spot

Magnitude: the size of a number, used for comparing numbers



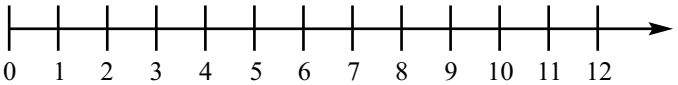
Professor K

As a child, Professor K was told that a picture was worth a thousand words. Pictures are ways to convey messages without words. Professor K discovered that mathematicians use special kinds of pictures. A ***graph*** is a type of mathematical picture. A graph is a drawing that shows a relationship between sets of numbers. Graphs, like people, come in all shapes. The most common graphs look like dots, lines, bars, or pies. In the newspaper you can see line graphs used to show the highs and lows of the ***stock market*** or ***commodities***. Pie graphs are in the shape of a circle and are most often used to show the percentage of income or expenses for a company, government, or family.

In this lesson, Professor K will show you how to read, interpret, and draw the different types of graphs.

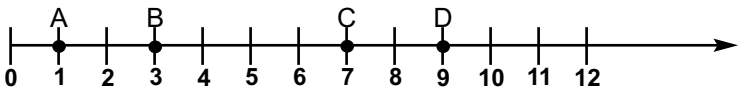
Number Lines

In previous lessons you have seen number lines. They are the simplest of all the graphs. A number line consists of a single horizontal line with ***tick marks*** for the position of each number in increasing ***magnitude*** from left to right as shown below.



The number line above has its origin at zero and increases to 12. The arrow at the right end signifies that the number line continues indefinitely.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Look at the number line above. The dot, or ***point*** A, represents a value of 1. Point B represents 3; point C represents 7; and point D represents 9.

THE CARTESIAN COORDINATE SYSTEM

The Cartesian coordinate system allows you to plot the exact location of a point in two dimensions. This type of plotting is especially important for the navigation of ships and planes. A graph of the Cartesian coordinate system consists of two number lines ***perpendicular*** to each other, or one ***horizontal*** line (***x-axis***) crossing (***intersecting***) one ***vertical*** line (***y-axis***).



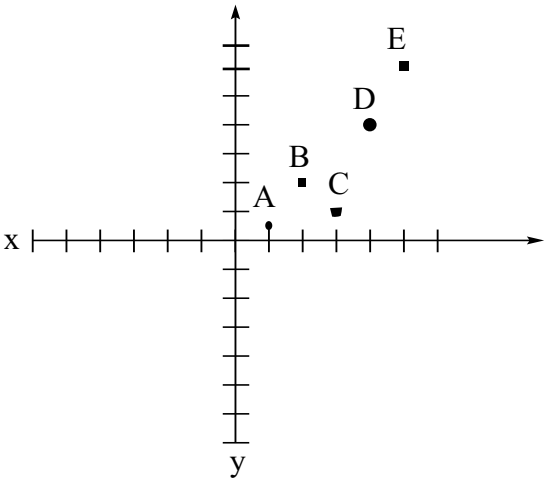


Figure 1.1

VOCABULARY

Point: a geometrical object that has no thickness or width, only location; a dot on a graph
Perpendicular: forming a right angle (90°) (—┼—)
Horizontal: sideways; a line drawn across the page left to right; the horizon is a horizontal line. (————)
X-axis: the horizontal base line in a Cartesian coordinate system; plural of axis is *axes* (ax-ēs)
Intersecting: crossing or meeting; two lines intersect at only one point
Vertical: perpendicular to the horizontal; directly overhead; a line drawn up and down on a page is a vertical line. (|)
Y-axis: the vertical base line in a Cartesian coordinate system
Ordered Pair: the name given a point in the Cartesian Coordinate System, denoted as (x, y)
Legend: a table used to explain a graph

The "name" a point is given is an ***ordered pair***. An ordered pair is denoted with a value corresponding to the x-axis and y-axis in this manner (x, y). The value of the number placed in the *x* position of the ordered pair is called the x-coordinate, and the value of the number in the *y* position is called the y-coordinate. The values of these coordinates are determined by following the vertical and horizontal lines that cross at each point back to the x-axis and y-axis. Pay close attention to the grid lines. Each line is not necessarily a whole number. Therefore, in the graph above, point A would have an ordered pair of (1, $\frac{1}{2}$), or one to the right of the y-axis and $\frac{1}{2}$ above the x-axis. The ordered pair corresponding to point B is (2, 2) or two to the right of the y-axis and two above the x-axis; to point C, (3, 1), or three to the right of the y-axis and one above the x-axis; to point D, (4, 4), or four to the right of the y-axis and four above the x-axis; and to point E, (5, 6), or five to the right of the y-axis and six above the x-axis.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Line Graphs in a Cartesian Coordinate System

Line graphs are drawn in a Cartesian coordinate system by merely connecting the points with a straight line. A line graph of the points A, B, C, D and E from the previous graph would look like this:

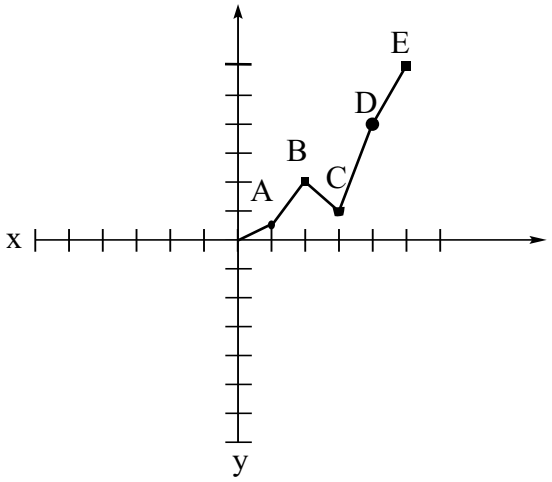


Figure 1.2
Line Graph

Bar and Column Graphs

Another type of graph is a *bar* or *column* graph. A graph of this type consists of a series of bars or columns that show the values of the x and y coordinates. Look at the same set of points from **Figures 1.1 and 1.2** in the column graph below. Notice that the columns are placed on the x-axis at the value of the x-coordinate and the top of the column stops at the value of the y-coordinate.

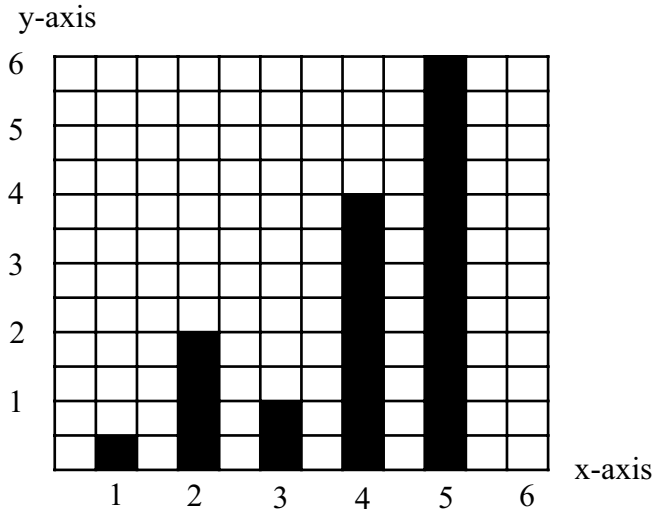


Figure 1.3
Column Graph

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Pie Graphs

Another type of graph is a *pie* graph. The name was chosen because the graph looks like a pie cut in slices. The total of all the individual percentages must equal 100%.

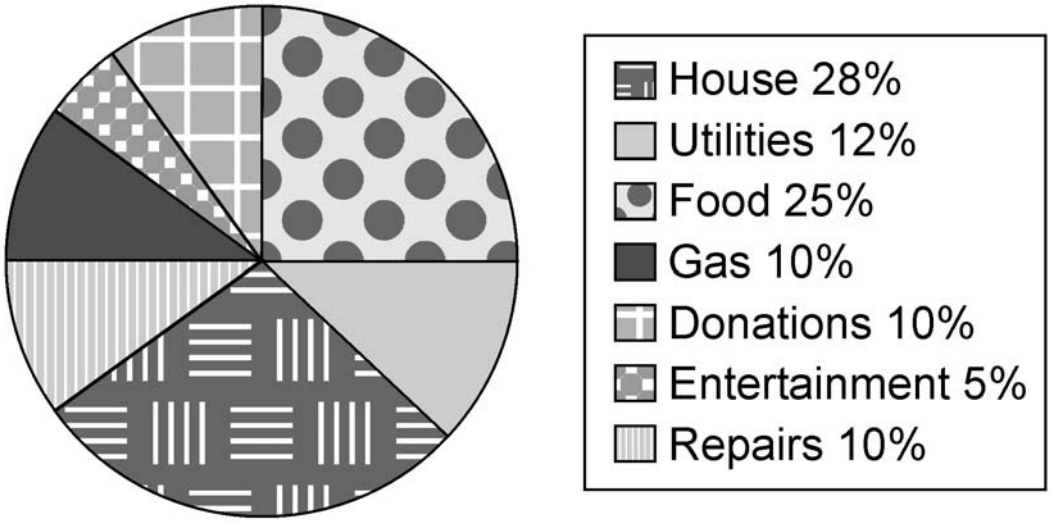


Figure 1.4
Pie Graph

A basic pie graph is pictured in **Figure 1.4**. Each section represents a different portion of a family's budget. The blocked area to the right of the graph is called the ***legend***. The legend of a graph tells what each portion of the graph represents and possibly the value of that portion.

Apply what you have learned about graphs in the activities for this lesson.

Life Principle



"Success is shooting at and hitting a noble goal."

—Anonymous

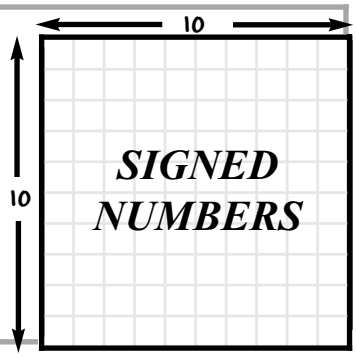
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 2

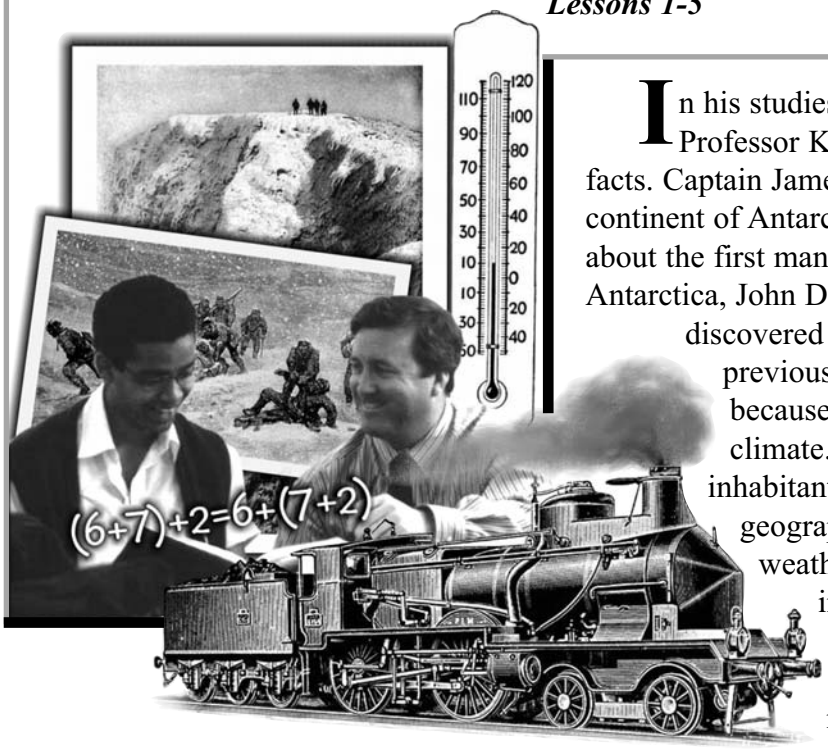
Signed Numbers

Chapter 3



Section 1

Lessons 1-5



In his studies of history and geography, Professor K noticed several interesting facts. Captain James Cook discovered the continent of Antarctica in 1773. Professor K read about the first man to land an expedition on Antarctica, John Davis in 1821. Professor K also discovered that unlike the Americas, no previous humans lived in Antarctica because of its severe cold and ***desolate*** climate. Today, the only human inhabitants are scientists who study the geography, geology, wildlife and weather of the region. Temperatures in the Antarctica are very cold. The lowest temperature, ever recorded on earth was recorded in Antarctica.

VOCABULARY

Desolate: wretched; gloomy
Positive number: a number located to the right of zero on a number line
Negative number: a number located to the left of zero on a number line
Meteorologist: a person who studies the weather
Mercury: a chemical element used in thermometers
Thermometer: a device used to determine temperature



Definition: Positive Number

A ***positive number*** is located to the right of zero on a number line. A positive number is designated by one of the following: +, $^{+}$, or no sign.

+1, $^{+}$1, or 1; +35, $^{+}$35, or 35; +246, $^{+}$246, or 246



Definition: Negative Number

A ***negative number*** is located to the left of zero on a number line. A negative number is designated by one of the following: -, or $^{-}$.

-9 or $^{-}$9; -85 or $^{-}$85; -754 or $^{-}$754

Professor K realizes that you too are probably familiar with cold temperatures. When listening to weather reports in the winter, you may hear a ***meteorologist*** report temperatures

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

such as ten degrees below zero or fifteen degrees below zero. You will remember from Chapter 2, that *zero* is considered the origin or starting place for numbers on a number line. Traditionally a ***thermometer*** is an upright (up and down) tube of glass filled with ***mercury***. The mercury rises and falls as the temperature changes. With zero as the origin and the thermometer standing upright, any temperature lower than zero would appear to be below or under zero. Temperatures below zero are represented by negative numbers. Temperatures above zero are represented by positive numbers. (Professor K shows a drawing of a thermometer in **Figure 2.1**.)

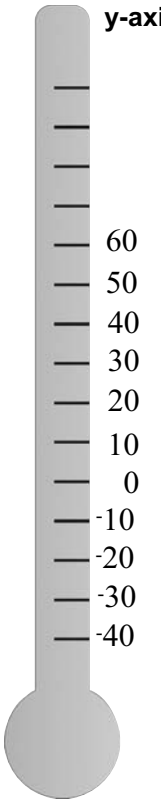


Figure 2.1

Looking at the thermometer in **Figure 2.1**, you can see that the gauge is a vertical number line (referred to as a y-axis). However, to understand negative numbers, this lesson will begin with negative numbers on the horizontal or x-axis. Look at the number line in **Figure 2.2**. Notice that the origin, *0*, is in the middle of the line segment. Positive numbers are to the right of *0* and, therefore, are *greater than 0*. Negative numbers are found to the left of and are *less than 0*. Zero is considered to be neither negative nor positive. Professor K reminds you that a number is greater than another number if the first number mentioned is to the right of the second number mentioned on a number line. Notice that the number line continues indefinitely in both directions as signified by the arrows on the end of each line.

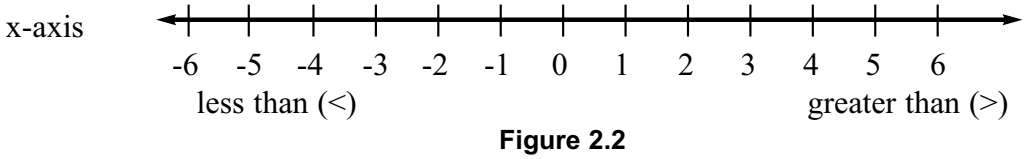


Figure 2.2

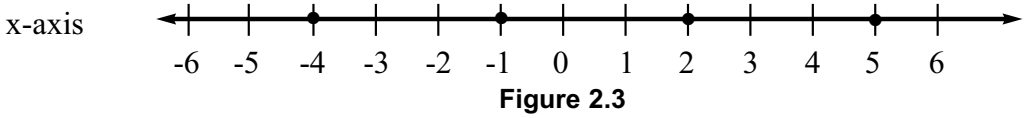


Figure 2.3

Look at **Figure 2.3**. The points shown are at -4, -1, 2, 5. The negative sign looks like a subtraction symbol, but the negative sign shows direction, not an operation. The negative sign means move to the left. The positive sign, +, means move to the right. $-4 < -1$ because -4 is to the left of -1. However, $2 > -1$ because 2 is to the right of -1. When considering numbers on a vertical number line, such as a thermometer (y-axis), negative numbers are below the origin, *0*, and the positive numbers are above the origin. Look again at the thermometer in **Figure 2.1**.

VOCABULARY

Altitude: distance above sea level; how high something is
Depth: how deep something is
Subtrahend: in a subtraction problem, the number being subtracted from another number



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

In a similar situation, measurements of ***altitude*** are calculated from sea level. Sea level is considered the origin. The height of a mountain is a certain number of feet above sea level. The ***depth*** of something in the ocean is measured similarly and represented as negative numbers. When something is below sea level, a negative number represents its altitude or depth in relationship to the origin, *sea level*.

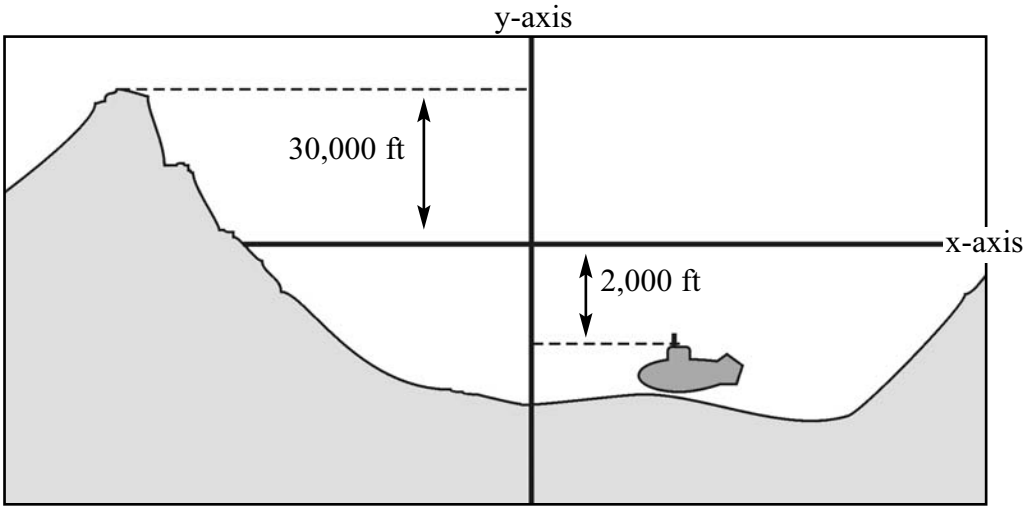


Figure 2.4

In **Figure 2.4**, the altitude or height of the mountain above sea level (bold line) is 30,000 ft. The depth of the submarine below sea level is 2,000 ft or -2,000 ft.

Notice how the Cartesian coordinate system is placed over the drawing in **Figure 2.4** so that you can understand the concept of negative numbers. **Figure 2.5** shows the Cartesian coordinate system related to the drawing in **Figure 2.4**.

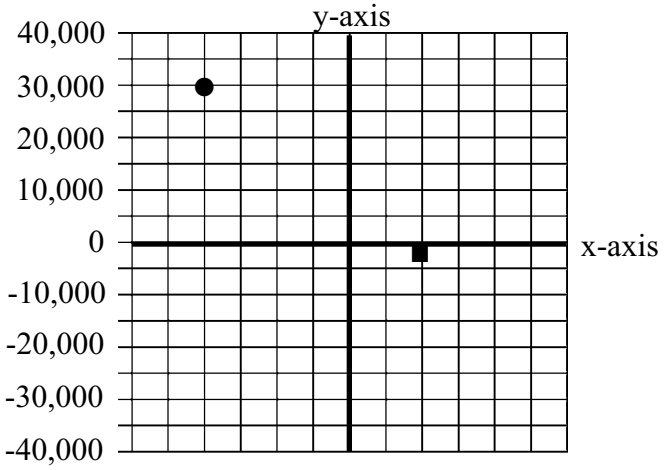


Figure 2.5

Notice in **Figure 2.5** that the point representing the height of the mountain (round dot) is plotted on the y-axis above *0* at the value, 30,000. The point representing the object under the water is plotted below the *0* on the y-axis at the value, -2,000 (square dot). The height and depth are plotted along the y-axis, because these two dimensions are vertical measurements. In this example the distance from *0* along the x-axis is not noted. However, navigators determine position or location using both coordinates. (The x-axis is the horizontal line that passes through *0* on the y-axis.)

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Addition and Subtraction Involving Negative Numbers

The addition and subtraction of numbers involving negative signs follow a few rules that help you understand and use numbers.



Rule 1

When two numbers of like sign are added together, simply add the numbers and the sign remains the same for the answer.

$^{+}3 + {}^{+}4 = {}^{+}7$ $\quad {}^{+}5 + {}^{+}7 = {}^{+}12$ $\quad {}^{-}2 + {}^{-}2 = {}^{-}4$ $\quad {}^{-}1 + {}^{-}5 = {}^{-}6$



Rule 2

When two numbers of opposite sign are added together, subtract the number of smaller magnitude from the number of larger magnitude and use the sign of the larger for the sign of the answer.

${}^{+}5 + {}^{-}7 \rightarrow 7 - 5 = 2 \rightarrow {}^{+}5 + {}^{-}7 = {}^{-}2$	The number of larger magnitude is $^{-}7$, therefore the sign of the answer is negative.
${}^{-}4 + {}^{+}8 \rightarrow 8 - 4 = 4 \rightarrow {}^{-}4 + {}^{+}8 = {}^{+}4$	The number of larger magnitude is +8, therefore the sign of the answer is positive.

You may have heard someone say that two wrongs do not make one right. In English class you were taught that using a double negative in a sentence makes a positive statement. The same is true in mathematics. In **Rule 3**, you see that two negatives make the number positive and that a negative and a positive make the number negative.



Rule 3

Subtraction of signed numbers is similar to addition of signed numbers. When subtracting a number, the operation of subtraction changes the sign of the ***subtrahend*** to the opposite sign and then follows the rule for addition.

${}^{+}2 - {}^{-}3 \rightarrow 2 + 3 = 5;$	$({}^{-}3 \rightarrow {}^{+}3$, then follow the rules for addition.)
${}^{-}4 - {}^{+}3 \rightarrow \quad {}^{-}4 + {}^{-}3 = -7;$	$({}^{-}3 \rightarrow {}^{+}3$, then follow the rules for addition.)

When adding positive and negative numbers think of walking forward and backward, or going up and down the rungs of a ladder. One step forward ($^{+}1$) and two steps back ($^{-}2$), places you one step back ($^{-}1$). ${}^{+}1 + {}^{-}2 = {}^{-}1$. Three steps forward ($^{+}3$) and one step back ($^{-}1$), places you two steps forward ($^{+}2$). ${}^{+}3 + {}^{-}1 = {}^{+}2$.

Apply what you have learned about negative numbers by working the activities for this lesson.

Life Principle



"Your attitude determines your altitude."
—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

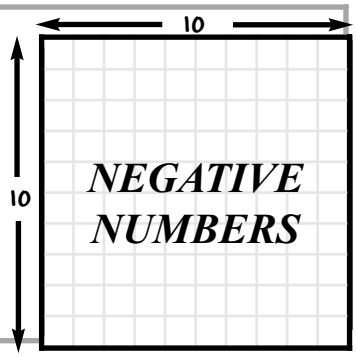
Basic Math Skills

-7

Signed Numbers

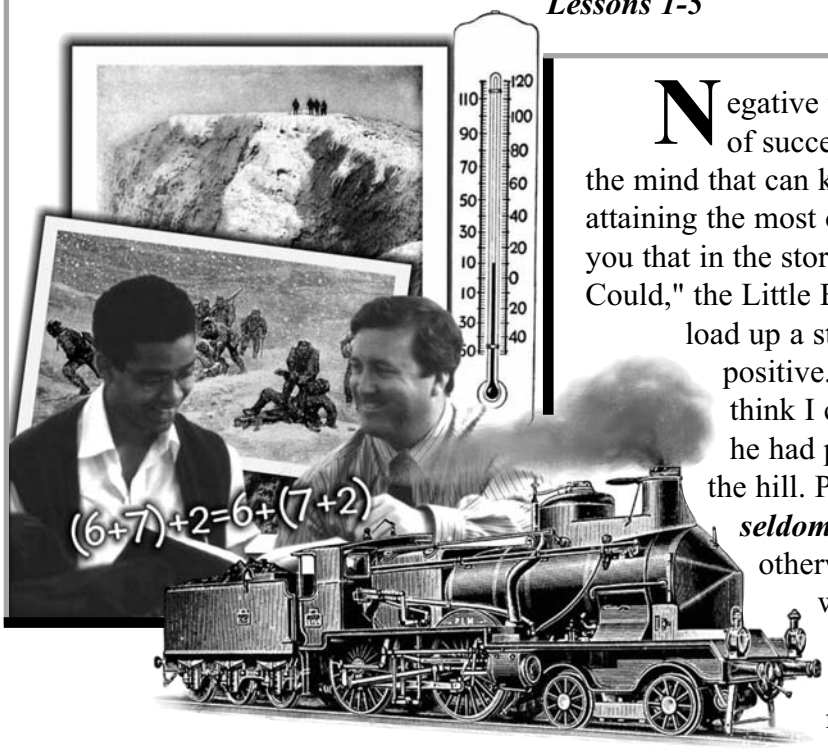
Lesson 3

Chapter 3



Section 1

Lessons 1-5



Negative thoughts or actions can rob you of success. ***Negativism*** is a "disease" of the mind that can keep an individual from attaining the most out of life. Professor K reminds you that in the story, "The Little Engine That Could," the Little Engine attempted to pull a large load up a steep hill. His thoughts were positive. He repeated over and over, "I think I can, I think I can," until finally he had pulled the load over the top of the hill. People with negative thoughts ***seldom*** accomplish what they otherwise could have accomplished with a positive outlook on problems or situations. Professor K knows that negative thoughts produce negative ***consequences***. Wise people learn to turn negative situations into positive results.

In mathematics, negative numbers have a positive use. In this lesson, Professor K will show you how negative numbers are used in fractions that are multiplied and divided.

Multiplication and Division with Fractions

Multiplication and division with fractions containing negative numbers follow certain rules as in addition and subtraction of whole numbers. Professor K knows that multiplication and division containing negative and positive numbers will be easier if you memorize the following rules.

Rule 1

When two numbers of like sign are multiplied or divided, simply perform the operation and place a positive sign in the answer.

VOCABULARY

Negativism: the attitude that life stinks and nothing good ever happens
Seldom: not very often
Consequences: results of our choices about people or events

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

positive × positive = positive;
$^{+}6 \times {}^{+}7 = {}^{+}42$; $^{+}4 \times {}^{+}3 = {}^{+}12$; (+ × + = +)

negative × negative = positive;
$^{-}2 \times {}^{-}4 = {}^{+}8$; $^{-}7 \times {}^{-}5 = {}^{+}35$ (- × - = +)

positive ÷ positive = positive;
$^{+}25 \div {}^{+}5 = {}^{+}5$ (+ ÷ + = +)

negative ÷ negative = positive;
$^{-}6 \div {}^{-}2 = {}^{+}3$ (- ÷ - = +)

RULE 2



When two numbers of opposite sign are multiplied or divided, the sign of the answer is negative.



negative × positive = negative;
$^{-}5 \times {}^{+}6 = {}^{-}30$ (- × + = -)

positive × negative = negative;
$^{+}2 \times {}^{-}9 = {}^{-}18$ (+ × - = -)

positive ÷ negative = negative;
$^{+}24 \div {}^{-}4 = {}^{-}6$ (+ ÷ - = -)

negative ÷ positive = negative;
$^{-}81 \div {}^{+}9 = {}^{-}9$ (- ÷ + = -)

RULE 3



Since fractions are another method of signifying a division problem, the following rules for the sign of the fraction apply when either or both the numerator and denominator are negative or positive numbers.

$$\frac{positive}{positive} = \text{positive} \qquad \frac{+2}{+3} = +\frac{2}{3} = \frac{2}{3}$$

$$\frac{negative}{negative} = \text{positive} \qquad \frac{-5}{-7} = +\frac{5}{7} = \frac{5}{7}$$

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

$$\frac{positive}{negative} = \text{negative} \qquad \frac{+7}{-8} = -\frac{7}{8}$$

$$\frac{negative}{positive} = \text{negative} \qquad \frac{-2}{+9} = -\frac{2}{9}$$

"Signed" fractions can be written in the following ways and still have the same value.

$$\frac{+4}{+5} = \frac{+4}{5} = \frac{4}{+5} = +\frac{4}{5} = \frac{4}{5} \qquad \frac{-5}{+9} = \frac{-5}{9} = \frac{5}{-9} = -\frac{5}{9}$$

$$\frac{+2}{+3} = \frac{+2}{3} = \frac{2}{+3} = +\frac{2}{3} = \frac{2}{3} \qquad \frac{-6}{+7} = \frac{-6}{7} = \frac{6}{-7} = -\frac{6}{7}$$

Multiplication, division and fraction problems may contain numbers with different signs. The following rule is a simplified version of Rules 1, 2, & 3 for determining the sign of the answer.



RULE 4

If there are an even number of negative signs, the answer is always positive.
If there are an odd number of negative signs, the answer is always negative.
If all the signs are positive signs, the answer is always positive.

To understand **Rule 4**, look at the following examples. You must count the number of negative signs in a multiplication or division problem to determine if the answer will be negative or positive.

a) $^{+}6 \times {}^{-}2 \times {}^{+}45 \times {}^{+}3 = {}^{-}1620$ a) negative; odd number of negative signs (-2)

b) $^{+}4 \div {}^{-}6 \times {}^{+}8 \times {}^{-}56 = {}^{+}298$ b) positive; even number of negative signs (-6),(-56)

c) $\frac{-5}{-7} = \frac{5}{7}$ c) positive; even number of negative of signs (-5),(-7)

d) $\frac{-4}{25} = -\frac{4}{25}$ d) negative; odd number of negative signs (-4)

Professor K knows that you can apply what you have learned about negative numbers by working the activities for this lesson.

LIFE PRINCIPLE



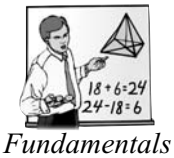
"By attempting the impossible we may obtain the possible."

—Italian Proverb

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



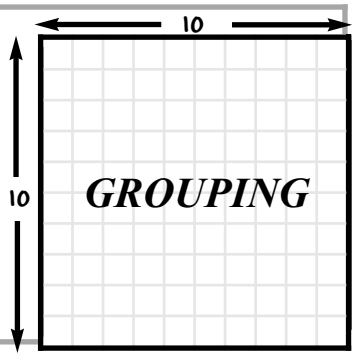
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Fundamentals

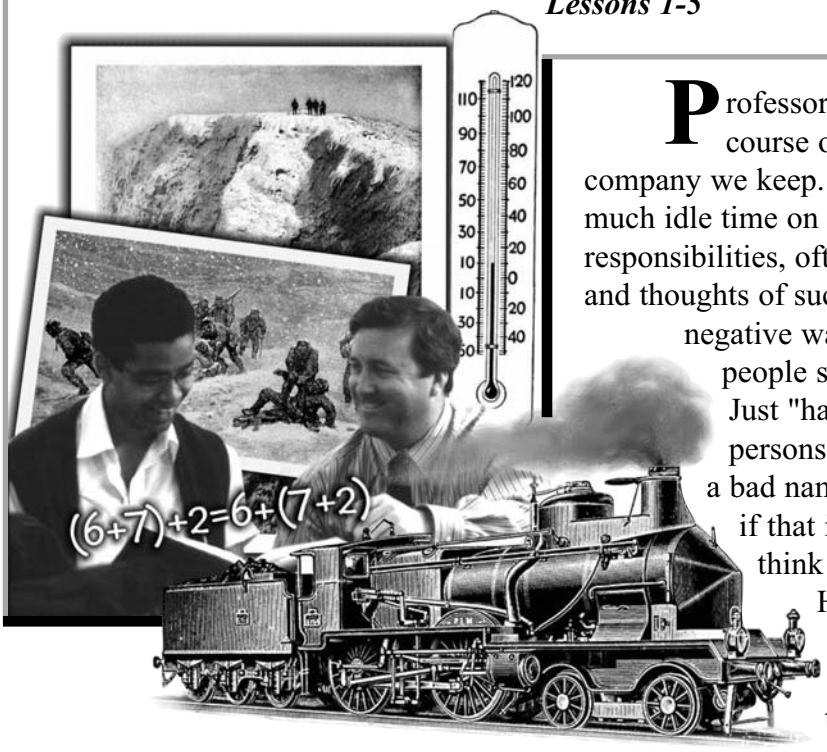
Lesson 4

Chapter 3



Section 1

Lessons 1-5



Professor K has observed that in the course of life we are judged by the company we keep. Groups of people with too much idle time on their hands with no positive responsibilities, often get into trouble. The actions and thoughts of such people influence others in negative ways. This is one reason so many people spend time in jails and prisons. Just "hanging out" with negative persons can cause an individual to "get a bad name" or negative reputation, even if that individual does not really act or think the way the group does. However, Professor K knows people who are concerned with occupying their minds and their time with wholesome work or entertainment. Positive people tend to accomplish more in life and influence others in positive ways. Positive people usually practice the values and morals necessary to succeed in life. Professor K surrounds himself with positive people.

Just like people, numbers in math problems often come in groups. How numbers are grouped in a math problem helps determine which operations are performed first. When solving large mathematical problems, ***grouping symbols*** are sometimes necessary to indicate which operation should be performed first. This lesson will present various grouping symbols and how they are used. A more in-depth study of the proper order of operations will be presented in Lesson 5.

When adding numbers together, two numbers are grouped and added together even if no grouping symbol exists. For the problem, 5 + 6 + 3 + 2, you must first add 5 + 6 to obtain 11. Then add 11 + 3 to obtain 14. Finally, add 14 + 2 to obtain the final answer of 16. All four numbers, 5, 6, 3,

VOCABULARY

Grouping symbols: symbols used to group numbers in a mathematical expression
Parentheses: a grouping symbol, ( )
Square brackets: a grouping symbol, [ ]
Braces or curly brackets: a grouping symbol, { }

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

and 2, were not added together in one step. One answer was obtained and then added to the next number, and that answer was added to the next and so on, until all the numbers were added. In some mathematical problems the order in which numbers are added together could be important. When the order of performing operations is important to the problem, mathematicians use grouping symbols. (Read **Rule 1**) The most common grouping symbol is the ***parentheses***. Professor K provides **Example Set 1** to show how *parentheses* are used to group numbers in different orders.



RULE 1

When a mathematical statement or equation contains grouping symbols, such as parentheses, the operations within the parentheses must be performed before any of the operations outside of the parentheses.



Example Set 1

a) $(1 + 3) + (5 + 4) + 4 \rightarrow 4 + 9 + 4 = 13 + 4 = 17$

a) This problem shows that *1 + 3* must be added, then *5 + 4*. The problem then becomes *4 + 9 + 4 = 13 + 4 = 17*.

b) $(3 + 3) - (2 + 2) \rightarrow 6 - 4 = 2$

b) For this problem, the additions within the parentheses must be performed first: *3 + 3 = 6* and *2 + 2 = 4*. Then the subtraction of those sums must be performed as shown.

c) $3 + 3 - 2 + 2 \rightarrow 6 - 2 + 2 \rightarrow 4 + 2 = 6$

c) Notice that this problem contains the same numbers as b) except for the parentheses. The answer is not the same as the previous answer, therefore, if *2* were the desired result as in part b), then the parentheses are necessary.

The next rule is one of the most helpful rules you will learn.



RULE 2

When a mathematical statement or equation contains two or more sets of grouping symbols, the operations in the innermost or "middle" set of grouping symbols are completed first.

Professor K recommends that you study **Example Set 2** until you develop a basic understanding of **Rule 2**.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 2

$4 + \{5+[3+(2 - 2)] + 1\}$

Step 1	$4 + \{5+[3+(2 - 2)] + 1\}$	Notice the three sets of grouping symbols. The parentheses, ( ), is the innermost set of grouping symbols. (Step 1)
Step 2	$4 + \{5 + [3 + 0] + 1\}$	First, the operation of subtraction is performed on the numbers within the parentheses as shown: *2 - 2* = 0. Now the ***square brackets*** are the innermost grouping symbol, therefore, the operation within the square brackets, [ ], is performed next. *3 + 0* = 3.
Step 3	$4 + \{5 + 3 + 1\}$	The only remaining grouping symbol is the ***braces*** or ***curly brackets***, { }, and the multiple operations of addition are performed within the curly brackets. *5 + 3 + 1* = 9.
Step 4	$4 + 9 = 13$	Step 4 yields the final answer of *13*.

OPERATION SYMBOLS

As you progress in your mathematical skills, other symbols are needed to indicate certain mathematical operations. Look at **Table 3.1** and study the new symbols for multiplication. Some of the symbols included are for review such as addition, subtraction, multiplication, and division. Professor K especially wants you to notice the new symbols for multiplication.

Table 3.1

addition	$+$
subtraction	$-$
multiplication	$\times, \bullet, (), [], \{\}, *$
division	$\div, \overline{)}$

In **Example Set 3,** Professor K shows how the new symbols to indicate multiplication are used and interpreted. Study this example closely to familiarize yourself with how the different symbols are used.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 3

a) $4 \times 6 = 4 \bullet 6 = 4\,(6) = 24$	a) This equation demonstrates that *4* multiplied by *6* can be written three different ways and the answer is the same.
b) $3 \times (5 \times 2) = 3(5 \bullet 2) = 3(10) = 30$	b) This equation demonstrates the use of two different multiplication symbols and Rule 1.
c) $2 \times [(6 \div 2) \times 2] = 2[(6 \div 2) \bullet 2] = 2[3 \bullet 2] = 2[6] = 12$	c) This equation is a demonstration of the use of different multiplication symbols and Rule 2.

In **Example Set 4**, Professor K demonstrates the use of several different operation and grouping symbols, and the use of **Rule 2**. Professor K reminds you to be sure to look back at **Table 3.1** and **Rule 2** as you look at **Example Set 4**.

Example Set 4

$9 + \{3(4 - 1) \div [(4 \bullet 2) + 1]\} =$

Step 1	$9 + \{3(4 - 1) \div [(4 \times 2) + 1]\} =$	Step 1 sets up the problem as shown with three different sets of grouping symbols and four different operation symbols.
Step 2	$9 + \{3(3) \div [8 + 1]\} =$	Step 2 uses **Rule 2** to begin solving the problem by performing the operations within the innermost grouping symbols first. *4 - 1* = 3, *4 • 2* = 8 and then *3(3)* = 9, and 8 + 1 = 9.
Step 3	$9 + \{9 \div 9\} =$	Step 2 yields the new equation in Step 3.
Step 4	$9 + 1 = 10$	In Step 4, *9 ÷ 9 = 1*, and *9 + 1 = 10*.

Apply what you have learned about grouping symbols and show Professor K you understand the concept of grouping by working the activities for this lesson.

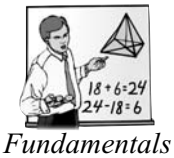
LIFE PRINCIPLE



"Birds of a feather flock together."
—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

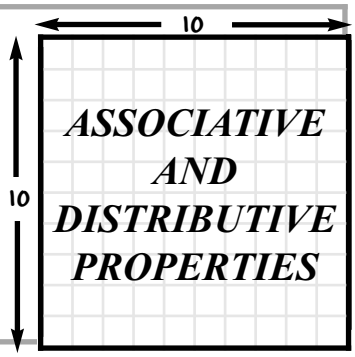
Basic Math Skills



Fundamentals

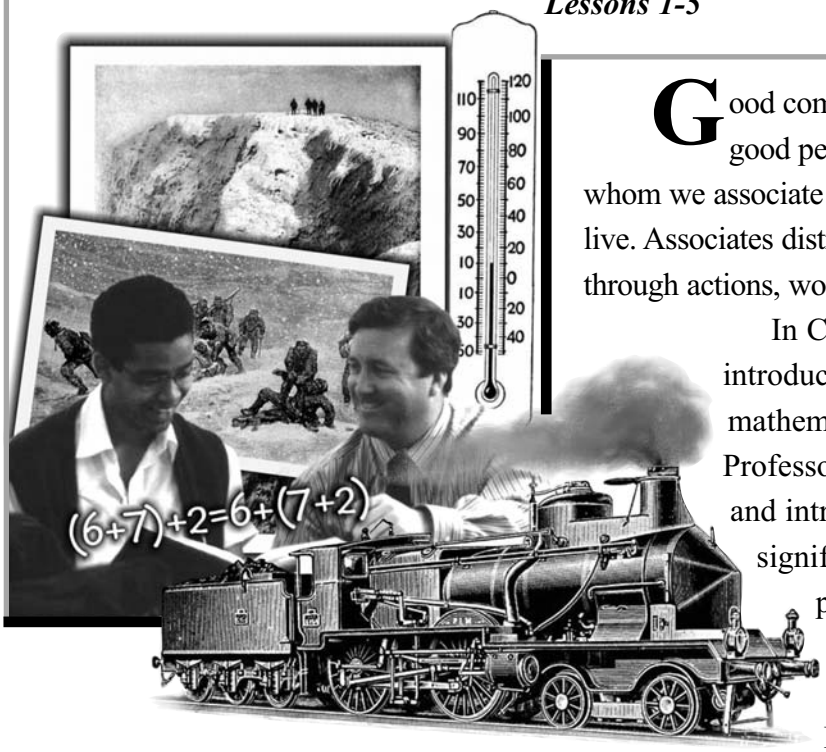
Lesson 5

Chapter 3



Section 1

Lessons 1-5





Professor K

Good companions help us become good people. Friends with whom we associate influence how we live. Associates distribute their influences through actions, words and values.

In Chapter 1, Professor K introduced you to some important mathematical ***properties***. In this lesson, Professor K will review one property and introduce another property of great significance for solving mathematical problems. The properties Professor K discusses in this lesson are the Associative Property and the Distributive Property. The principles of grouping symbols and the various symbols of multiplication learned in Lesson 4 are important to both the Associative Property and the Distributive Property. Remember, how numbers are grouped or associated together can change the outcome of a problem just as our associates determine the outcome of our lives.

The Associative Property of Addition from Chapter 1 is restated below for your convenience.

VOCABULARY

Properties: specific pieces of math rules that allow you to work Algebra
Process: application of a property to solve an equation
Signify: indicate; point out

Associative Property

When three or more numbers are added, the order in which they are added does not matter.

$$(6 + 7) + 2 = 6 + (7 + 2)$$
$$4 + (3 + 2) = (4 + 3) + 2$$

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Grouping symbols are important when using the Associative Property, as the grouping symbols signify what numbers and operations are grouped together to be worked first. Another mathematical property that occurs often is the Distributive Property. Professor K recommends that you learn the ***process*** used in the Distributive Property as this will help you greatly as you increase your mathematical skills.



Distributive Property of Multiplication

The Distributive Property of Multiplication is defined by the following mathematical statements:

Addition: $2(3 + 4) = (2 \cdot 3) + (2 \cdot 4) = 6 + 8 = 14$

Subtraction: $3(6 - 5) = (3 \cdot 6) - (3 \cdot 5) = 18 - 15 = 3$

In words, the Distributive Property states that when a certain number is multiplied by numbers within a grouping symbol, the certain number is "distributed" by multiplication to each of the numbers within the grouping symbol with the operation of addition or subtraction remaining between the products.

In **Example Set 1** below, notice how Professor K distributes the numbers outside the parentheses by multiplication "across" the addition and subtraction symbols.

Example Set 1

a) $2(4 + 5) = (2 \cdot 4) + (2 \cdot 5) = 8 + 10 = 18$
b) $4(5 - 3) = (4 \cdot 5) - (4 \cdot 3) = 20 - 12 = 8$
c) $2(4 + 3 + 5 - 1) = (2 \cdot 4) + (2 \cdot 3) + (2 \cdot 5) - (2 \cdot 1) = 8 + 6 + 10 - 2 = 22$

In Chapter 2, Professor K told you about other civilizations using letters to represent numbers. Mathematicians today use letters to represent a number for which they do not know a value.



Definition: Unknown

An "unknown" is a letter used to ***signify*** the position of the answer in a mathematical statement or equation.



Professor K offers **Example Set 2** to familiarize you with unknowns.

Example Set 2

Name the unknown in each equation.

a) $x + 1 = 1$	a) x
b) $z - 5 = 5$	b) z
c) $2(y) = 6$	c) y
d) $x \div 8 = 2$	d) x



Definition: Variable

A variable is a letter used to represent any number, usually in a mathematical definition, property, or formula.

As stated in the definition, a *variable* represents any number, therefore, Professor K restates the following Associative and Distributive Properties in a more generalized form.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Associative Property of Addition

Let a, b and c be any number; then the Associative Property states that:

$$(a + b) + c = a + (b + c)$$

Distributive Property of Multiplication

Let a, b and c be any number; then the Distributive Property of Multiplication is defined by the following mathematical statements:

Addition: $a(b + c) = (a \bullet b) + (a \bullet c)$

Subtraction: $a(b - c) = (a \bullet b) - (a \bullet c)$

In words, the Distributive Property states that when a certain number, "a", is multiplied by numbers, b and c, within a grouping symbol, the number, "a" is "distributed" by multiplication to each of the numbers, b and c, within the grouping symbol. In the above example of the Distributive Property, notice the two products $(a \bullet b)$ and $(a \bullet c)$. The operations of addition or subtraction (**-**) remain between the products.

Apply what you have learned about the Associative and Distributive Properties by working the activities for this lesson. Professor K reminds you that after completing the activity for this lesson you may ask your teacher for Quiz 1. If you make an 85% or higher on all three quizzes for Chapter 3 you may be exempt from taking the chapter test.

Life Principle



"You are judged by the company you keep."

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

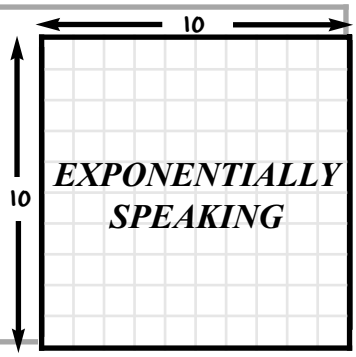
Basic Math Skills



Lesson 6

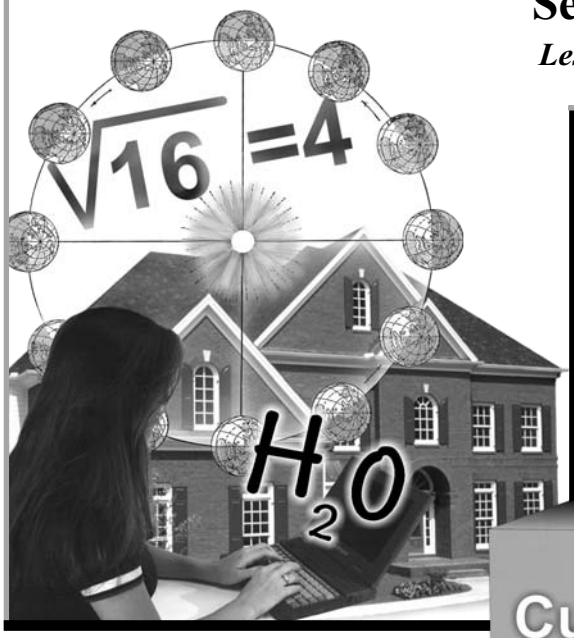
Exponents

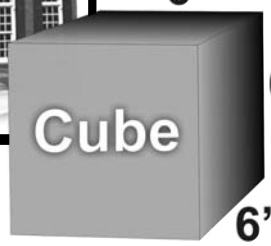
Chapter 3



Section 2

Lessons 6-10





Professor K once served on a jury in the District Court in his hometown. As the testimony was presented, Professor K noticed a man typing on a typewriter in front of the judge. The man did not seem to be typing in a hurried manner or the same as most typists. This man was a ***court reporter***. In the courts of our country, a court reporter keeps a written record of legal proceedings. The typewriter the court reporter uses does not have the same keys as regular typewriters or computers. The court reporter does not have to type every letter of words; the keys on his special typewriter are a form of ***shorthand.*** For many years, secretaries used shorthand to take ***dictation*** from bosses, then typed the full texts of the letters. Now, bosses use hand held recorders for dictation. Professor K remembered that mathematicians, too, have developed many shorthand methods for relating math problems to other mathematicians. Professor K reminds you that multiplication is a shortcut to addition. The problem, 3 + 3 + 3 can be written, 3 × 3. Therefore, 3 × 3 is a shorthand method of adding three, three times. Both expressions equal 9.

Professor K introduces another shorthand method called ***exponents***. An exponent is a small number placed to the upper right of the ***base***. The base is the number to be used as a ***factor*** and the exponent is the number of times the base is used as a factor. For example, 3 × 3 becomes 3 squared (3^2). *3* is the *base*, and *2* is the exponent.

When a number is written with an exponent, the number is raised to the ***power*** of the value of the exponent. In the previous example, *3* to the second power is the same as three squared (3^2). In **Table 6.1** Professor K shows examples of 2 raised to different powers. He also shows how the

VOCABULARY

Court reporter: officer of the court who keeps written records of all court proceedings
Shorthand: method of writing faster with fewer letters and words
Dictation: reading or speaking aloud for the purpose of reproduction in writing
Exponent: a small superscripted (raised) number to the right of a base number
Base: a number that is raised to a power
Factor: numbers multiplied together

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

numbers are written in ***exponential form***, and he translates the numbers into words so you will be able to understand the concept of exponents.

Table 6.1

base	exponent or power	exponential form	words
2	2	$2^2 = 2 \times 2 = 4$	two squared
2	3	$2^3 = 2 \times 2 \times 2 = 8$	two cubed
2	4	$2^4 = 2 \times 2 \times 2 \times 2 = 16$	two to the fourth power
2	5	$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$	two to the fifth power
2	6	$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$	two to the sixth power

Example Set 1 shows how Professor K raises other numbers to various powers.

Example Set 1

$3^3 = 3 \times 3 \times 3 = 27$

$3^4 = 3 \times 3 \times 3 \times 3 = 81$

$4^2 = 4 \times 4 = 16$

$4^3 = 4 \times 4 \times 4 = 64$

$4^4 = 4 \times 4 \times 4 \times 4 = 256$

$1^{10} = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

$(-2)^3 = (-2) \bullet (-2) \bullet (-2) = (4) \bullet (-2) = -8$

$(-5)^2 = (-5) \bullet (-5) = 25$

$0^7 = 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$



Professor K points out that in **Example Set 1**, when the base is negative (-2) and the exponent is an odd number, (1, 3, 5, 7, etc.) the answer is negative (-8). When the base is negative (-5) and the exponent is an even number (2, 4, 6, 8, etc.), the answer is positive (25).

Another important observation Professor K believes you can see in **Example Set 1** is that *1* raised to any power always produces *1* as the answer ($1^{10} = 1$), and *0* raised to any positive power is always *0* ($0^7 = 0$).

Professor K now presents several important properties concerning exponents. Be sure to study these properties as well as the examples for each.

VOCABULARY

Power: to use a number as a factor, the number of times equal to the value of the exponent
Exponential form: a number written with an exponent
Reciprocal: result of inverting a fraction

Property of Fractions with Exponents

When a fraction is raised to an exponent, both the numerator and denominator are raised to that exponent.

$$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2 \bullet 2}{3 \bullet 3} = \frac{4}{9}$$

$$\left(\frac{-1}{5}\right)^3 = \frac{-1}{5} \times \frac{-1}{5} \times \frac{-1}{5} = \frac{(-1) \bullet (-1) \bullet (-1)}{5 \bullet 5 \bullet 5} = \frac{-1}{125}$$

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Property of the Exponent of Zero

Any number raised to the power of zero is one.



$3^0 = 1;\ 8^0 = 1;\ 45^0 = 1;\ 3456^0 = 1;\ (\frac{15}{53})^0 = 1$

Property of the Exponent of One

Any number raised to the power of one is the base number.

$3^1 = 3;\quad 8^1 = 8;\quad 45^1 = 45;\ 3456^1 = 3456;\ (\frac{15}{17})^1 = \frac{15}{17}$

Property of Negative Exponents

Any number raised to a negative power changes the number to its ***reciprocal*** raised to the positive power.
Consider the number 3 raised to a power of -2 : 3^{-2}.
If the number is a whole number, make it into a fraction by placing it over one [$3 \rightarrow {}^3/_1$] then invert the fraction to find the reciprocal [${}^3/_1$ inverts to become ${}^1/_3$].
After finding the reciprocal, use the positive of the exponent $[{}^1/_3]^2$.

$3^{-2} = (\frac{1}{3})^2$

$4^{-3} = (\frac{1}{4})^3$

Apply what you have learned about exponents by working the activities for this lesson.

Life Principle



"Nobody can go back and start a new beginning, but anyone can start today and make a new ending."

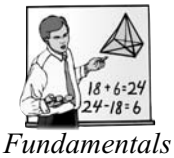
—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

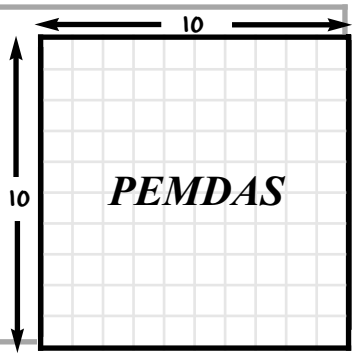
Basic Math Skills



Lesson 7

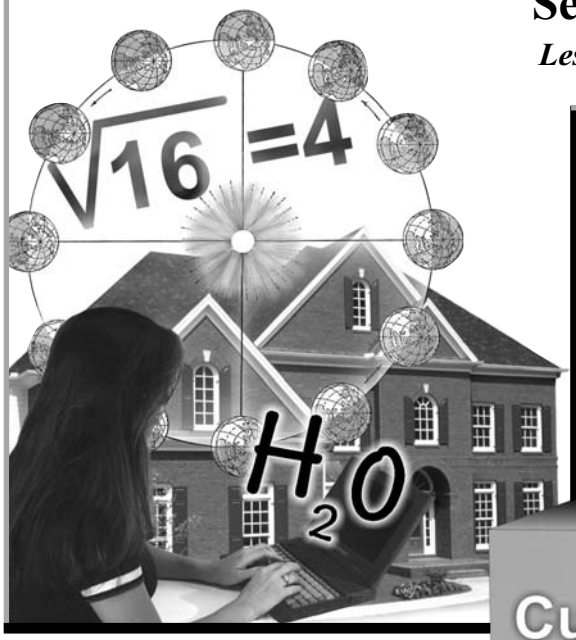
Fundamentals

Chapter 3

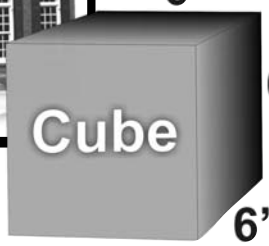


Section 2

Lessons 6-10



In Professor K's studies, he noticed that everything in the universe reflects *design* and *order*. Planets revolve around the sun in an orderly fashion. The periodic chart of the elements reflects order that allows predictability. Governments and their citizens are controlled by rules and regulations that promote order. Professor K also knows that when relationships are conducted in the right order and follow proper principles and procedures, social problems that are encountered work out much easier.



Mathematics has order or procedures, also. This order in mathematics is called the ***order of operations***. The order of operations describes the steps for working any arithmetic or algebraic problem. Professor K will present the order of operations in this lesson and he will show you the proper steps for solving mathematical problems.

You must know the meaning of a few new words so that you can follow the explanation of the Order of Operations. Study the following definitions presented by Professor K and then study the examples to increase your understanding.

VOCABULARY

Order of operations: a commonly accepted procedure mathematicians use to solve problems

Term: part of a mathematical expression; often indicates a product and may contain any number of factors of that product

Mathematical expression: two or more terms separated by addition or subtraction symbols

Exponential term: a term containing an exponent



Definition: Term

A ***term*** is a part of a ***mathematical expression***. A term usually consists of a product of two or more factors. However, a term may be one number or letter.

4	3(3)
$2 \bullet x$	$3 \bullet 6 \bullet 1$

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Definition: Exponential Term

An ***exponential term*** is a term that contains at least one exponent.



5^3 $\quad$ $3(3^2)$

$2 \bullet x^5$ $\quad$ $3^2 \bullet 4^3 \bullet 5^4$

Definition: Mathematical Expression

A mathematical expression is a combination of two or more terms separated by an addition or subtraction symbol.

$4 + 3(2)$ $\quad$ $2(6) - 9(34) + 16$

$2 \bullet x - 3 + 45 \div 5$

Order of Operations

The following order should always be followed when solving mathematical problems.

Parentheses (grouping symbols); ( ), [ ], { } — Perform all indicated operations within grouping symbols first. The expression inside the innermost grouping symbols must be completed first.

Exponents — Find the value of all numbers with exponents. $3^2 = 3 \times 3 = 9$

Multiplication: / **D**ivision: — Perform all multiplication or division first, left to right as you come to them. $3 \times 3 = 9$; $4 \div 2 = 2$

Addition: / **S**ubtraction: — Perform all addition and subtraction, left to right as you come to them. $5 + 3 = 8$ $\quad$ $8 - 2 = 6$

A method of remembering the proper sequence in the *Order of Operations* used by Professor K is to remember the following saying.

Please Excuse My Dear Aunt Sally (PEMDAS).

a	x	u	i	d	u
r	p	l	v	d	b
e	o	t	i	i	t
n	n	i	s	t	r
t	e	p	i	i	a
h	n	l	o	o	c
e	t	i	n	n	t
s	s	c			i
e		a			o
s		t			n
		i			
		o			
		n			

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Professor K has derived PEMDAS from the first letter of each word in the *Order of Operations*.

Example Set 1

Step 1	$4 + (3 \bullet 2 - 4) - 2^2$	Step 1 states the problem. Following the *Order of Operations*, look inside the parentheses first. The multiplication, *3 • 2*, is performed first. (PEMDAS: multiplication comes before subtraction)
Step 2	$4 + (6 - 4) - 2^2$	In Step 2, you still are working inside the parentheses. Then 3 • 2 = *6*, yielding the new mathematical expression within the parentheses: *6 - 4*.
Step 3	$4 + (2) - 2^2$	Working within the parentheses in Step 3, we see that *6 - 4 = 2*. Following the *Order of Operations*, PEMDAS, we have finished with the P (parentheses) and now move on to the E (exponents).
Step 4	$4 + 2 - (2 \times 2) \rightarrow 4 + 2 - 4$	In Step 4, the exponential term, 2^2, becomes 2×2. The multiplication is performed, yielding $2 \times 2 = 4$. The problem is now *4 + 2 - 4*.
Step 5	$(4 + 2) - 4 = 6 - 4 = 2$	In Step 5, using PEMDAS: left to right, *4 + 2 = 6*, and *6 - 4 = 2*. (Parentheses added to show the first addition as you move left to right of *4 + 2*.)

Example Set 2

Step 1	$5 + 6(3 - 2^3) - 1 + 2[2(4 + 1)]$	In Step 1, the problem is stated. Look at the term furthest to the right, *2[2(4 + 1)]*. PEMDAS forces you to work with this term first.
Step 2	$5 + 6(3 - 2^3) - 1 + 2[2(5)]$	In the term, *2[2(4 + 1)]*, you see two sets of grouping symbols, [ ] and ( ). PEMDAS requires you to work within the innermost set of grouping symbols first, therefore, *4 + 1* = 5. (See Step 2.)
Step 3	$5 + 6(3 - 2^3) - 1 + 2[10]$	Continue working with the last term within the square brackets, *2(5)* = 10. (See Step 3.)

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 4	$5 + 6(3 - 2^3) - 1 + 2[10]$ $5 + 6(3 - (2 \times 2 \times 2)) - 1 + 2[10]$ $5 + 6(3 - 8) - 1 + 2[10]$	The last term is multiplication only, with just a number in the parentheses. Therefore, following PEMDAS you must go to the second term, which includes operations within the parentheses. In Step 4, $2^3 = 2 \times 2 \times 2 = 8$. Even within the parentheses you must follow PEMDAS.
Step 5	$5 + 6(-5) - 1 + 2[10]$ $5 + (-30) - 1 + 20$	Within the parentheses, $3 - 8 = -5$. Then according to PEMDAS, you must do all multiplications as you come to them left to right, therefore, $6(-5) = 6 \bullet -5 = -30$ and $2[10] = 2 \bullet 10 = 20$. (See Step 5.)
Step 6	$5 + (-30) -1 + 20 \rightarrow$ $5 - 30 -1 + 20$	Step 6 demonstrates that $+ (-30) \rightarrow -30$ (positive • negative = negative).
Step 7	$-25 - 1 + 20 = -26 + 20 = -6$	In Step 6 the only operations remaining are addition and subtraction. $5 - 30 = -25$; $-25 - 1 = -26$; and finally $-26 + 20 = -6$ in Step 7.

Professor K points out in Step 6 of **Example Set 2**, the statement was made that + (-30) = -30. This statement is based on the following principle.



PRINCIPLE

When no number exists immediately in front of a grouping symbol, the number is understood to be the number 1.



$$(4 + 3) = 1(4 + 3) = 1(7) = 1 \bullet 7 = 7$$
$$- (2 + 3) = -1(2 + 3) = -1(5) = -1 \bullet 5 = -5$$
$$+ (6 - 9) = +1(6 - 9) = +1(-3) = +1 \bullet (-3) = -3$$

Show your understanding of the *Order of Operations* to Professor K by working the activities for this lesson.

LIFE PRINCIPLE



"The hardest thing to find is a person who can concentrate on what is important."

—Henry Doherty

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

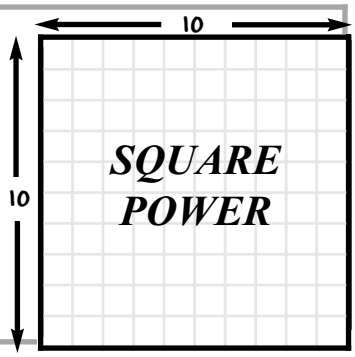
Basic Math Skills



Lesson 8

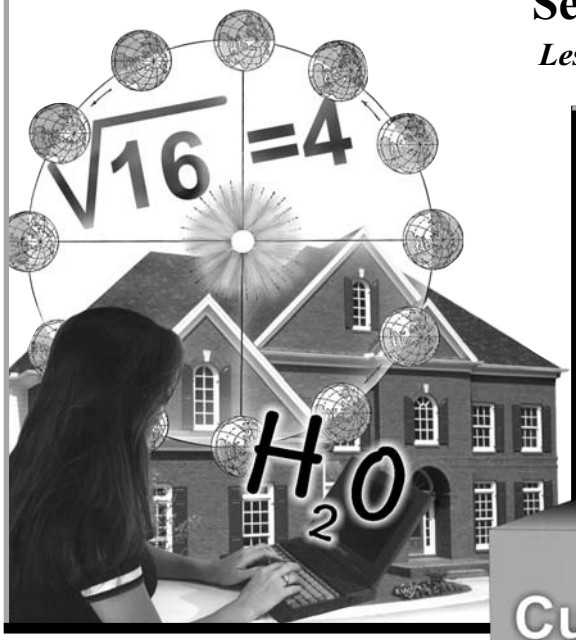
Exponents

Chapter 3

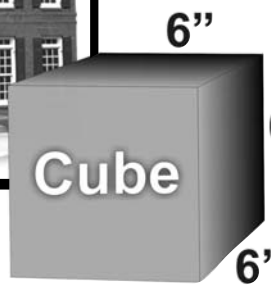


Section 2

Lessons 6-10



In Lesson 6, you studied *exponents*. Some of the exponential terms were given special names, such as *four squared* ($4^2 = 4 \times 4 = 16$) and *three cubed* ($3^3 = 3 \times 3 \times 3 = 27$). Professor K reminds you that to raise a number to a power, means to use the number as a factor the same number of times as the value of the exponent.



When Professor K and his wife married, one primary goal for them was to achieve the "American Dream" of owning a home. Professor K's wife had a large part in the planning of the house, while Professor K participated in building the house, or at least closely supervised the construction of it. Professor K wanted to make sure that his wife was pleased and that the house reflected beauty and security.

In planning the home, an ***architect*** had drawn the plans and determined the ***area*** of the house in *square feet*. The volume of concrete for the foundation was measured in *cubic yards*. The builder had to know the amount of concrete required for the foundation in order to complete the house.

Square of a Number

Finding the area of something, such as a house, is based on the geometric figure of a ***square***. Hence, we use the term, *square feet*. If the water heater closet of a house is 4 ft wide and 4 ft long, then the closet is in the form of a square and the area is determined by multiplying *4* times *4*; ($4\text{ft} \times 4\text{ft} = 16\text{ft}^2$) as in **Figure 8.1**. In words, the water heater closet is four feet by four feet, so the area is *sixteen square feet*. That statement means that *16 squares* with sides of length *one foot* each are contained in the area of the water heater closet as shown in **Figure 8.1**.

VOCABULARY

Architect: one who plans or designs buildings

Area, A: $A = s^2$, where s is the length of one side of a square; generally length, l, times width, w; $A = l \bullet w$

Square of a number: to square a number means to multiply it by itself one time. $4^2 = 4 \bullet 4$

Square: a geometric figure with all sides (s) equal, and all angles equal to 90°.

Rectangle: a geometric figure with opposite sides equal, and all angles equal to 90°.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

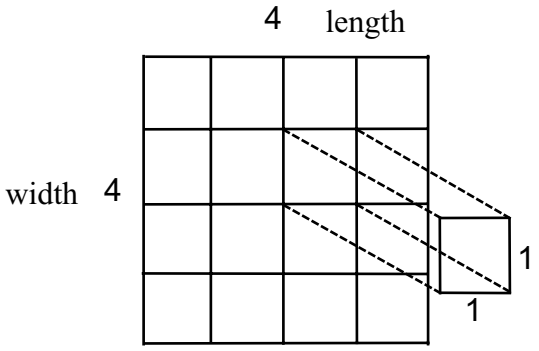




Figure 8.1

Example Set 1

Follow these steps with Professor K to find the following areas:

a) What is the area of a room, 10 ft on each side?

$10^2 = 10 \text{ ft} \times 10 \text{ ft} = 100 \text{ sq ft}$

a) The room is *10* feet on each side, therefore the area is determined by multiplying 10×10 or 10^2. The area is *100* square feet.

b) What is the area of a building 35 ft on each wall?

$35^2 = 35 \text{ ft} \times 35 \text{ ft} = 1225 \text{ sq ft}$

b) The building is *35* feet on each wall. The area is determined by multiplying 35×35 or 35^2. Therefore the area is *1225* square feet.

In **Example Set 1**, Professor K squared the numbers, because the sides of the figures described had four sides of equal length. Professor K determined the area of these figures by multiplying two sides together (s^2), or since the figures were squares with equal sides, Professor K found the area by squaring one side ($A = s^2$).

The area of figures with unequal sides is found in a similar manner. The area is determined by multiplying the value of the shorter side (width) by the value of the longer side, (length). The area is given in square units since the area consists of a series of squares. Look at **Figure 8.2** to understand how Professor K carries the concept of area over into another geometric figure.

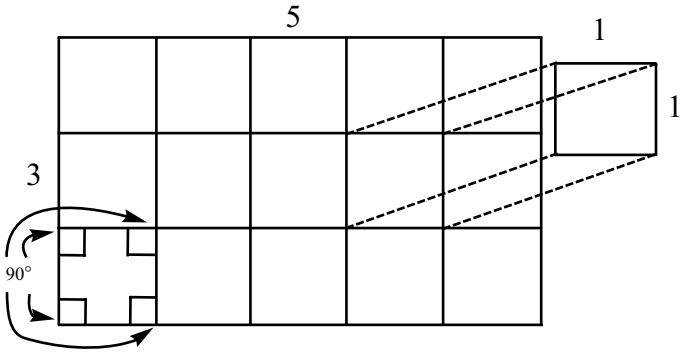


Figure 8.2

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

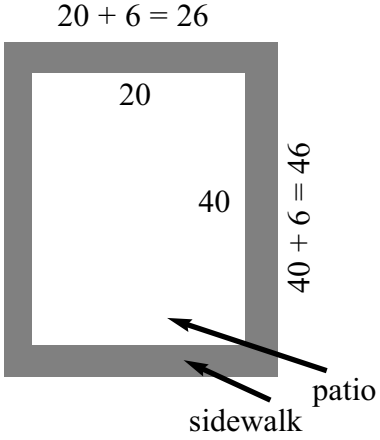
Notice in **Figure 8.2**, the width is *3* units and the length is *5* units. The area of the figure consists of fifteen squares with sides of one unit. The geometric figure represented in **Figure 8.2** is known as a ***rectangle***, because the opposite sides are equal and all the angles are 90°. The area, A, of the rectangle in **Figure 8.2** is: $A = l \times w = 3$ units $\times$ 5 units = 15 square units.



Often for various reasons, we need to know the area of "borders." Look at **Example Set 2** to see how finding the area of two rectangles is necessary for determining the area of a border.

Example Set 2

Professor K wishes to have a 3 ft wide sidewalk around his patio. The patio is in the shape of a rectangle of length, 40 ft, and width, 20 ft. What is the area of the sidewalk?



Step 1

The first step to solving any problem is to understand what is being asked. To do this, it is often helpful to draw a picture or diagram. The figure to the left represents the patio with the sidewalk around it.

Step 2

The outer width of sidewalk = width of patio + two widths of the sidewalk = 20 + 3 + 3 = 26

The outer length of the sidewalk = length of the patio + two widths of the sidewalk = 40 + 3 + 3 = 46

Notice that the drawing consists of two rectangles. The dimensions of the inner rectangle, the patio, are *20 ft* × *40 ft*. The dimensions of the outer boundary of the sidewalk are *26 ft* × *46 ft*. These dimensions are determined by adding two widths of the sidewalk to the dimensions of the patio, because the sidewalk is on all sides of the patio.

Step 3

Area of outer rectangle:
$A_o = 46 \times 26 = 1196$
Area of the inner rectangle:
$A_i = 20 \times 40 = 800$

Find the area of each rectangle.

Step 4

Area of the sidewalk:
$A_s = A_o - A_i =$
1196 sq ft - 800 sq ft = 396 sq ft

Find the area of the sidewalk by subtracting the area of the smaller rectangle from the area of the larger rectangle.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Show Professor K that you understand the principle of *squaring* numbers by working the activities for this lesson.

Life Principle



"You get married not to be happy, but to make each other happy."

—Roy L. Smith

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

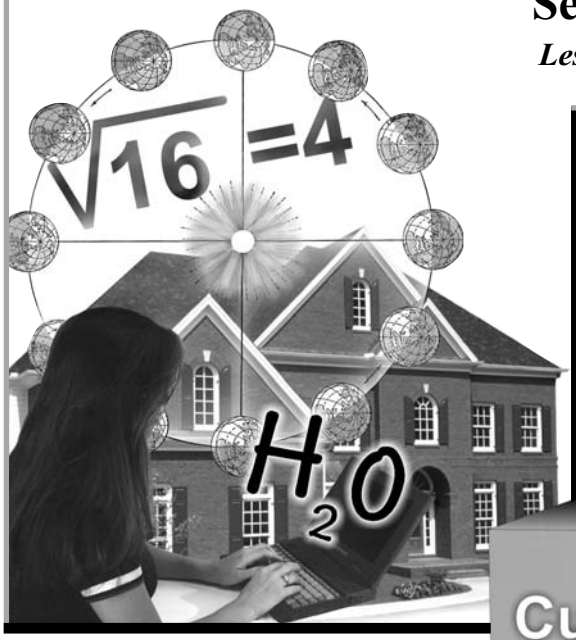
Chapter 3

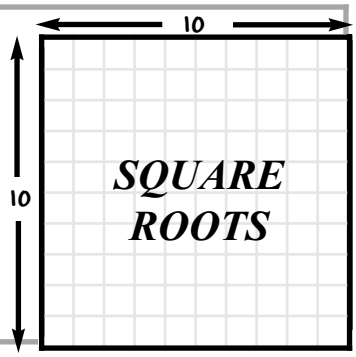
Lesson 9

Radicals

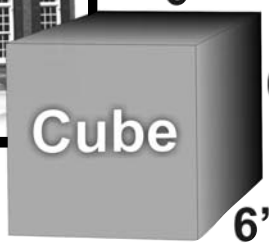
Section 2

Lessons 6-10





In Lesson 8, Professor K explained *squaring* a number. In this lesson Professor K will put his mathematical car in reverse, as he did with division, and calculate the ***square root*** of numbers. Sometimes the total area (square of a number) is known, but the length of a side of the square that makes up that area is needed. Professor K reminds you that to raise a number to a power means to use the number as a factor, the same number of times as the value of the exponent ($4^2 = 4 \times 4 = 16$). To determine the square root of sixteen, Professor K must see that 4^2 equals sixteen. By dropping the exponent, ^{2}, Professor K can calculate that the *square root* of *16* is *4*.



Square Root

To find the square root of a number is the opposite operation from squaring a number. Square root means to reverse or "undo" the square of that number.

$4^2 = 4 \times 4 = 16$ → *4* is the square root of *16*, also written, $\sqrt{16} = 4$

$5^2 = 5 \times 5 = 25$ → *5* is the square root of *25*, also written, $\sqrt{25} = 5$

If the area of the patio in **Example Set 2** in the previous lesson was 400 square feet, and the patio was in the shape of a square instead of a rectangle, you could find the length of a side of the patio by using *square root*. The *square root* of 400 square feet is 20 feet because 20 feet × 20 feet = 400 square feet. (See definition of square root.)

VOCABULARY

Square root: one of the equal factors of a squared number. The number $16 = 4^2 = 4 \cdot 4$; therefore, 4 is the square root of 16.

Perfect square: a whole number square root of a number, having no remainder; also the principal square root of a number

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Square roots are used to find the length of the side of a square if the area is known. Look at **Example Set 1** as Professor K works square root problems so you can understand this process more completely.

Example Set 1

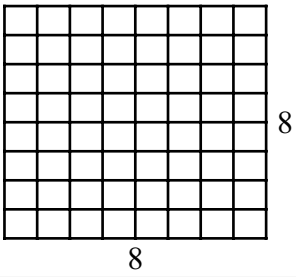
a) $\sqrt{9} = 3$

a) *3* is the square root of *9*, because 3×3 or $3^2 = 9$.

b) $\sqrt{36} = 6$

b) *6* is the square root of 36, because 6×6 or $6^2 = 36$.

c) If the area of a square is 64, what is the length of one side of the square?



c) Since the area is 64, to find the length of one side of the square, you must "take the *square root*" of *64*. This operation yields the equation: $\sqrt{64} = 8$. *8* then is the length of one side of a square, because 8×8 or $8^2 = 64$. *8* is also the length of all the sides since the figure is a square and all the sides of a square are equal.

The squares discussed in **Example Set 1** were ***perfect squares***, or their *square roots* were a whole number with no remainder. Below is a table of the most useful perfect squares. Professor K encourages you to memorize them as you did multiplication tables. This will help you solve *square root* problems more quickly and easily.

Table 9.1

$\sqrt{1} = 1$	$\sqrt{25} = 5$	$\sqrt{81} = 9$	$\sqrt{169} = 13$
$\sqrt{4} = 2$	$\sqrt{36} = 6$	$\sqrt{100} = 10$	$\sqrt{196} = 14$
$\sqrt{9} = 3$	$\sqrt{49} = 7$	$\sqrt{121} = 11$	$\sqrt{225} = 15$
$\sqrt{16} = 4$	$\sqrt{64} = 8$	$\sqrt{144} = 12$	$\sqrt{256} = 16$



DEFINITION: PRINCIPAL SQUARE ROOT

The principal *square root* of a number is "*a*" if $a^2 = b$, where "*a*" is a nonnegative number.



The *square roots* in the table above are the principal *square roots* of each number, because they are positive. *Square roots* can also be negative ($-5 \bullet -5 = 25$). The negative *square roots* will be discussed in later lessons.

Some numbers must be split into other factors, one of which is a perfect square so that the *square root* of the perfect square can be determined and a remainder is written in radical form (a number containing a radical sign).

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Look at **Example Set 2** as Professor K shows you how to take the *square root* of numbers that are not perfect squares.

Example Set 2

a) $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2 \cdot \sqrt{3} = 2\sqrt{3}$

a) *12* is split up into the factors, *4* • *3* because *4* is a perfect square (2 × 2 = 4). The *square root* of *4* equals *2*. The *square root* of *3* is not a perfect square so it is written in radical form. The final answer becomes $2\sqrt{3}$.

b) $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4 \cdot \sqrt{2} = 4\sqrt{2}$

b) *32* is split up into the factors, *16* • *2* because *16* is a perfect square (4 × 4 = 16). The *square root* of *16* equals *4*. The *square root* of *2* is not a perfect square so it is written in radical form. The final answer becomes $4\sqrt{2}$.

To determine the *square root* of a large number you must split the number into as many factors as you can that are perfect squares and then multiply their *square roots* together. Any remainder that is not a perfect square is placed under the radical sign of the answer as in **Example Set 2**.



Look at **Example Set 3** as Professor K demonstrates how to calculate the *square root* of a large number using this process.

Example Set 3

a) $\sqrt{128} = \sqrt{4 \bullet 4 \bullet 4 \bullet 2} = \sqrt{4} \bullet \sqrt{4} \bullet \sqrt{4} \bullet \sqrt{2} = 2 \bullet 2 \bullet 2 \bullet \sqrt{2} = 8\sqrt{2}$

a) *128* is split up into the factors, *4* • *4* • *4* • *2* because *4* is a *perfect square* (2 × 2 = 4). *4* is a factor of *128* three times. The *square root* of *4* equals 2. The *square root* of *2* is not a perfect square, so it is written in radical form. After multiplying *2* • *2* • *2*, the final answer becomes $8\sqrt{2}$.

b) $\sqrt{300} = \sqrt{25 \bullet 4 \bullet 3} = \sqrt{25} \bullet \sqrt{4} \bullet \sqrt{3} = 5 \bullet 2 \bullet \sqrt{3} = 10\sqrt{3}$

b) *300* is split up into the factors, *25* • *4* • *3* because *25* is a perfect square (5 × 5 = 25) as is *4*. The *square root* of *25* equals *5*. The *square root* of *4* is *2*. The *square root* of *3* is not a perfect square so it is written in radical form. After multiplying *5* • *2*, the final answer becomes $10\sqrt{3}$.

Show Professor K that you understand the principle of square roots by working the activities for this lesson.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

LIFE PRINCIPLE



"Problems are easier when we get to the root of the problems."

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

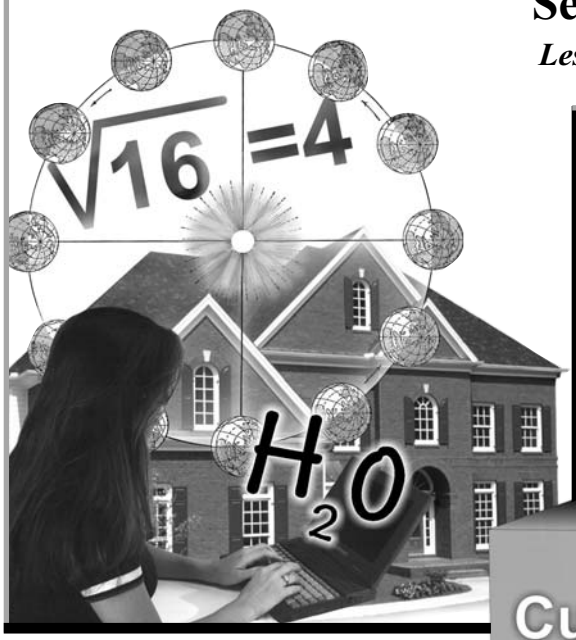
Chapter 3

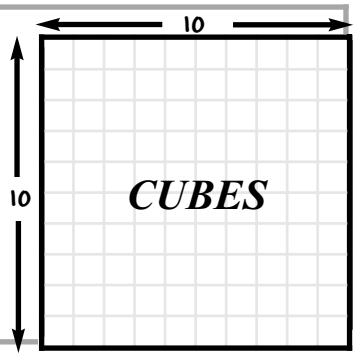
Lesson 10

Exponents

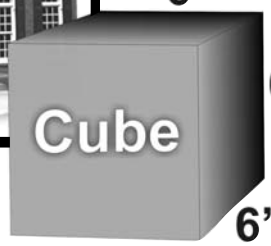
Section 2

Lessons 6-10





In the previous lessons, you studied squares and *square roots*. In this lesson Professor K will teach you about ***cubes***. Professor K reminds you that to raise a number to a power means to use the number as a factor the same number of times as the value of the exponent. Therefore, to find the square of a number you must raise the number to the power of 2, or find the product of the number times itself (number times number). Similarly, to find the ***cube of a number***, *x*, you must raise the number to the power of 3, or find the product of the number three times itself (number times number times number). Squares are used in relationship to area. Cubes are used in relationship to ***volume***. Look at **Example Set 1** as Professor K shows you how to cube a number and how volume is determined in cubic units.



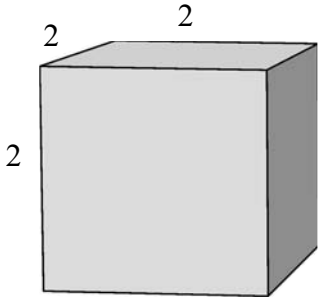
VOCABULARY

Cube: a geometric figure with all edges equal, all angles equal to 90°, and six faces or sides, each in the shape of a square.
Cube of a number: a number is cubed when that number is taken as a factor three times
Volume, V: $V = s^3$, where *s* is the length of one edge of a cube; generally length, *l*, times width, *w*, times height, *h*
$V = l \times w \times h$

Example Set 1

Find the following volumes:

a) The volume of a box, 2 ft on each edge.



a) The box is *2* feet on each edge, therefore, the volume is determined by multiplying $2 \times 2 \times 2$ or 2^3 ($2^3 = 2$ ft $\times$ 2 ft $\times$ 2 ft = 8 cubic ft). The volume is 8 cubic feet.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

b) The dimensions of a room are 10 ft long, 10 ft wide, and 10 ft high. An air conditioning repairman must determine the volume of the air in the room, so he can determine the size of air conditioning system needed to keep the room cool.

b) The room is *10* feet along each edge. The volume is determined by multiplying $10 \times 10 \times 10$ or 10^3 (10^3 = 10 ft × 10 ft × 10 ft = 1000 cubic ft). Therefore, the volume is 1000 cubic feet.

In **Example Set 1**, the numbers Professor K used were cubed, or raised to the third power, because the figures described were in the shape of cubes (boxes with all sides of equal length). When the volume of a figure with edges of unequal length is found, the volume is also given in cubic units. Professor K reminds you that square units of measurements are used as a standard for determining area. Area is determined by a series of squares of one unit on each side placed side by side. In a similar manner, cubic units of measurement are used for determining volume. Volume is determined by stacking a series of cubes of length one unit on each edge on top of each other. Another way Professor K explains volume is to fill the space with cubes of one unit length on each edge. If you stacked cubes of one unit in length, width and height, three wide, three deep and three high you would fill a space of 27 cubic units.

In general, *volume* equals *length* times *width* times *height*, or $V = l \bullet w \bullet h$. Multiplying the three dimensions (length, width and height) together yields the volume of the object.

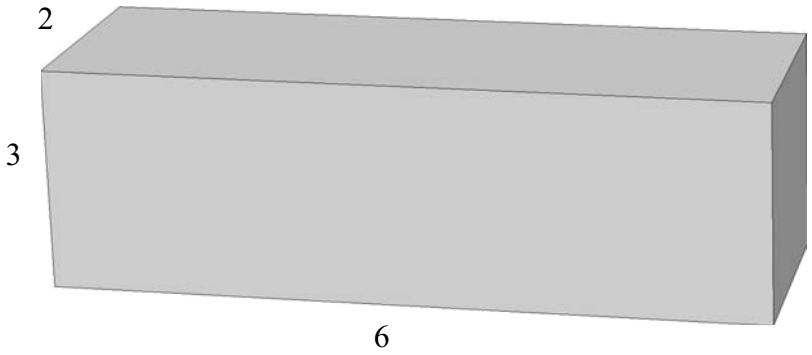


Figure 10.2

Professor K wants you to notice that in **Figure 10.2**, the width is 2 units, the length is 6 units, and the height is 3 units. The volume of the figure could be represented by thirty-six, 36, cubes with edges of one unit stacked side-by-side and on top of each other. The volume, V, of the geometric shape in **Figure 10.2** is: $V = l \bullet w \bullet h$ = 6 units × 2 units × 3 units = 36 cubic units.

Example Set 2

Professor K wants to ship some books he purchased on his trip to Europe. Professor K measured each book and determined that the amount of space all the books together occupied was 29 cubic feet. Assuming that the books would fit tightly in the crate, would a crate 4 ft × 3 ft × 2 ft be large enough to ship all the books? Let's help Professor K decide.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Step 1 Read and reread the problem to determine that Professor K wants to ship some books home from Europe. He needs a crate that will hold at least 29 cubic feet. Professor K has a crate that is 4 ft × 3 ft × 2 ft. Professor K must determine if the crate will hold all the books.

Step 2 Professor K must perform the calculation of $V = 4 \text{ ft} \times 3 \text{ ft} \times 2 \text{ ft} = 24$ cubic feet.

Step 3 Professor K compares the volume of the crate with the volume needed and realizes that since 24 is less than 29 ($24 < 29$) the books will not fit in the crate and he must find a bigger crate.

Show Professor K your understanding of volume by completing the activities for this lesson.

LIFE PRINCIPLE



"Fill up your life with good and there will be no room for bad."

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

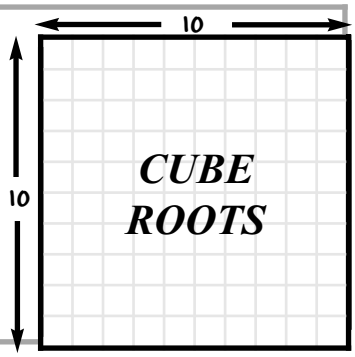
Basic Math Skills



Radicals

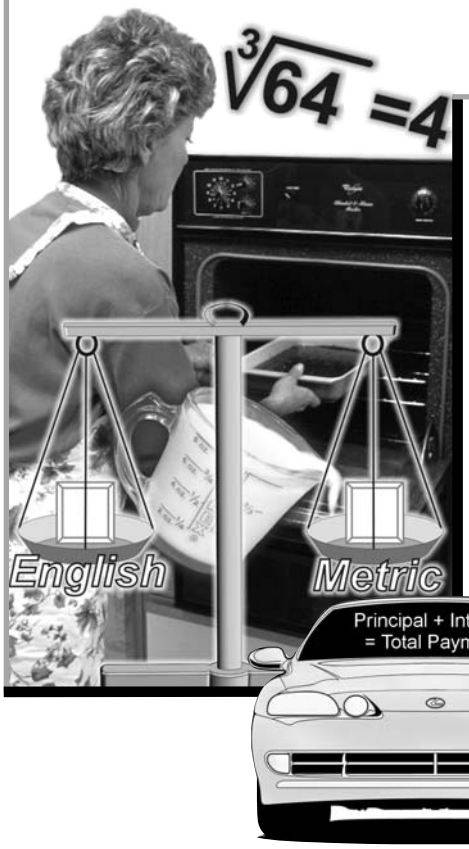
Lesson 11

Chapter 3



Section 3

Lessons 11-15



In a previous lesson, you studied *square roots*. In this lesson, Professor K will teach you about *cube roots*. Professor K reminds you that to determine the *square root* of a number you must find a number that when multiplied times itself yields the number for which the *square root* is being determined. (The square root of 4 = 2 because 2 × 2 = 4.) Therefore, to find the square root of a number you must determine what number is raised to the second power, or find the equal factors of x^2 ($x \bullet x$). Similarly, to find the ***cube root*** of a number, x^3, you must determine the three equal factors of x raised to the third power ($x \bullet x \bullet x$). Professor K reminds you that just as squares are used in relationship to area; ***cubes*** are used in relationship to volume. Professor K states the definition of *cube root* below.



Professor K

DEFINITION: CUBE ROOT OF A NUMBER

The *cube root* of a number, *a*, is a number, *b*, if $b^3 = a$.

VOCABULARY

Cube root: The cube root of a number, x, is b, if $b^3 = x$.
Cube: a geometric figure with all edges equal, all angles equal to 90° and six faces, each the shape of a square.
Index: a superscript (small number), written to the upper left of a radical sign, to indicate the desired root, as in $\sqrt[3]{64}$

- $4^3 = 64$, therefore *4* is the *cube root* of *64*, also written, $\sqrt[3]{64} = 4$
- $5^3 = 125$, therefore *5* is the *cube root* of *125*, also written, $\sqrt[3]{125} = 5$
- $(-3)^3 = -27$, therefore *-3* is the *cube root* of *-27* also written, $\sqrt[3]{-27} = -3$

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Professor K wants you to notice that in the definition of *cube root,* the *cube root* of a negative number is negative. The answer is negative because the *cube root* is one of three equal factors of a number. As shown in the definition, $(-3)^3 = (-3)\bullet(-3)\bullet(-3) = (9)\bullet(-3) = -27$, therefore, since -3 is one of three equal factors of -27, -3 is the *cube root* of -27.

As he did with *square roots* in Lesson 9, Professor K lists some of the most common *cube roots* in the following table. Professor K suggests that you memorize this table to determine *cube roots* more quickly.

Table 11.1

$\sqrt[3]{1} = 1$	$\sqrt[3]{-8} = -2$	$\sqrt[3]{64} = 4$	$\sqrt[3]{-125} = -5$
$\sqrt[3]{-1} = -1$	$\sqrt[3]{27} = 3$	$\sqrt[3]{-64} = -4$	$\sqrt[3]{216} = 6$
$\sqrt[3]{8} = 2$	$\sqrt[3]{-27} = -3$	$\sqrt[3]{125} = 5$	$\sqrt[3]{-216} = -6$

Cube roots are used to find the length of the edge of a cube if the volume is known. Look at **Example Set 1** as Professor K works through several cube root problems.

Example Set 1

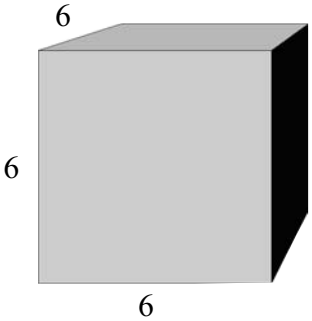
a) $\sqrt[3]{8} = 2$

a) The *cube root* of *8* is *2*, because $2 \times 2 \times 2 = 2^3 = 8$. *2* is an equal factor of *8* three times, therefore, *2* is the *cube root* of *8*.

b) $\sqrt[3]{-1} = -1$

b) *-1* is the *cube root* of *-1*, because $-1 \times -1 \times -1$ or $(-1)^3 = -1$.

c) If the volume of a cube is 216 cubic inches, what is the length of one edge of the cube?



c) Since the volume is *216* cubic inches, to find the length of one edge of the cube, you must "take the *cube root*" of *216*. This operation yields the equation: $\sqrt[3]{216} = 6$. *6* then is the length of one edge of the cube, because $6 \times 6 \times 6$ or $6^3 = 216$. *6* is also the length of all the edges since the figure is a cube and all the edges of a cube are equal.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

The numbers in **Table 11.1** were perfect cubes, or in other words the *cube roots* were positive or negative whole numbers with no remainders. As with *square roots*, you can determine the *cube root* of any larger number by factoring the number and then grouping the three equal factors together. Then multiply the roots together and place any remaining factors in the answer within the radical sign.

Look at **Example Set 2** as Professor K demonstrates this procedure.

Example Set 2

a) $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{(2 \cdot 2 \cdot 2) \cdot 2} = 2 \cdot \sqrt[3]{2}$

a) *16* is split up into the factors, *8 • 2* because *8* is a perfect cube (2 × 2 × 2 = 8). The *cube root* of *8* equals *2*. The *cube root* of *2* is not a perfect cube, so it is written in radical form. The final answer becomes $2\sqrt[3]{2}$. Notice that Professor K grouped the three factors of *8* together to emphasize the grouping of like factors to determine *cube roots*.

b) $\sqrt[3]{648} = \sqrt[3]{324 \cdot 2} = \sqrt[3]{162 \cdot 2 \cdot 2} = \sqrt[3]{81 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{27 \cdot 3 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{9 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2} = \sqrt[3]{(3 \cdot 3 \cdot 3) \cdot 3 \cdot (2 \cdot 2 \cdot 2)} = 3 \cdot 2\sqrt[3]{3} = 6\sqrt[3]{3}$

b) *648* is split up into the factors, as shown. Professor K then groups equal factors in groups of three to determine *cube roots*. He then places those roots outside the radical sign and multiples *3 • 2* to obtain *6*. The *cube root* of *3* is not a perfect cube so it is written in radical form. The final answer becomes $6\sqrt[3]{3}$.

Professor K wants you to show your understanding of *cube roots* by working the activity for this lesson.

Life Principle



"Every problem has a root cause. Find it, fix it, and life will be much easier."

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

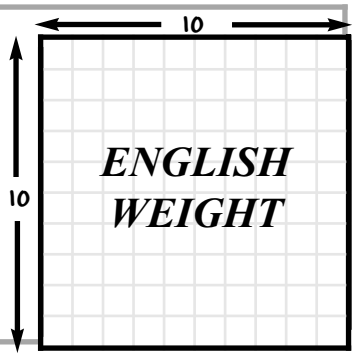
Basic Math Skills



Converting

Lesson 12

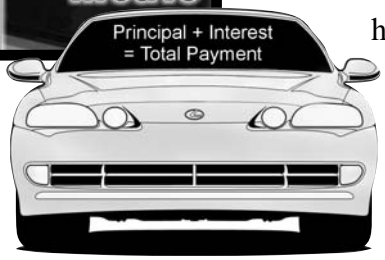
Chapter 3



Section 3

Lessons 11-15







Professor K

Professor K reminds you that in previous chapters you learned about measures of *length* and *volume*. Another important measurement used in the world involves ***weight***. Everyday you purchase and use products of different weights. Previously, Professor K discussed how length and volume can be expressed in either the English or Metric System of measurement. In this lesson Professor K will show you how *units of weight* are expressed in the *English System*. In the next lesson Professor K will show you how to convert weights within the Metric System.

Table 12.1

Weight - English System

1 pound, lb	=	16 ***ounces***, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

VOCABULARY

Weight: the amount of gravitational force exerted on an object

Ounce: basic unit of weight in the English system, abbreviated *oz*

Conversions: changes from one unit of measure to another unit of measure; pounds to ounces; tons to pounds, etc

Look at **Table 12.1**. Professor K points out that to change units in the left column to units in the right column, you must apply *multiplication*. When converting units from the right column to the left column, you must apply *division*. Professor K has set up **Table 12.2** to show this procedure more clearly.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Table 12.2

Multiply			Divide		
pounds, lb	→	ounce, oz	pound, lb	←	ounce, oz
cwt	→	pound, lbs	cwt	←	pounds, lbs
ton	→	cwt	ton	←	cwt

By looking at the following examples, you can see how Professor K makes ***conversions*** in the English System.

Example Set 1

Step 1	4800 oz = ?? cwt	To convert or change *4800* oz into one hundred weight (cwt), apply conversions from **Table 12.1** that shows 16 oz = 1 lb. With this fact in mind, the question of "How many one hundred weights are in *4800* ounces?" is answered by division. (See Step 1.)
Step 2	4800 oz ÷ 16 oz = 300 lbs	To change *4800* ounces into *hundred weight*, you must divide *4800* oz by *16* oz. This means 4800 oz = *300* lbs. (See Step 2.)
Step 3	300 lbs ÷ 100 lbs = 3 cwt	The next step is to divide *300* pounds by *100* pounds to see how many hundred weights are in *300* pounds. The answer is *3* cwt. (See Step 3.)

Example Set 2

Step 1	2 tons = ?? oz	To convert *tons* to *ounces*, refer again to the facts in **Table 12.1**. (See Step 1.)
Step 2	2 tons × 2000 lbs = ?? lb	Since *2000* pounds = *1* ton, *2* tons will be multiplied by *2000*. Therefore, *2* tons = 4000 lbs. (See Step 2.)
Step 3	4000 lbs × 16 oz = ?? oz	Since 16 ounces = 1 lb, *4000* pounds will be multiplied by *16*. This means *4000* pounds = *64,000* ounces. (See Step 3.)
Step 4	2 tons = 64,000 oz	Therefore, *2* tons = *64,000* ounces. (See Step 4.)

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 3

Step 1	256 oz = ?? lbs	To convert *ounces* to *pounds*, you must again refer to the facts in **Table 12.1**. (See Step 1.)
Step 2	256 oz ÷ 16 oz = ?? lb	Since 16 ounces = 1 pound, *256* ounces will be divided by *16*. (See Step 2.) Performing the calculation yields *256 ÷ 16 = 16*.
Step 3	256 oz = 16 lbs	Therefore, *256* oz = *16* lbs. (See Step 3.)

You are making good progress. You are gaining knowledge and confidence applying basic math skills. Now apply these skills by completing the activities for this lesson.

LIFE PRINCIPLE



"Be responsible for yourself and your actions. Take pride in what you do."

—Arvin Boote

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



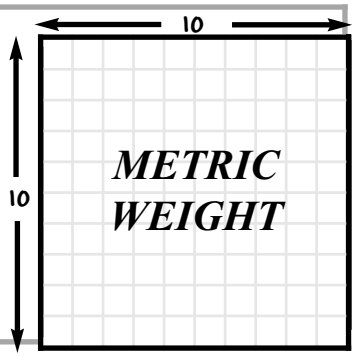
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Lesson 13

Converting

Chapter 3



Section 3

Lessons 11-15



On Professor K's family vacation to Europe, his children noticed that other countries also used the Metric System to show ***weight*** as well as length and volume. In the previous lesson Professor K demonstrated conversions of weight within the English System of measurement. In this lesson, Professor K will explain the *Metric System* of weight measurement.

Professor K reminds you that the Metric System is used to measure length and volume. The Metric System is also used to measure weight. The definition for *weight* in the Metric System is the same as the definition for *weight* in the English System.

However, the basic unit of weight in the Metric System is the ***gram***. To describe different weights in the metric system, different prefixes, or "word beginnings," are added to *gram*, the basic unit for weight. **Table 13.1** shows you these prefixes, their abbreviations and their meanings. You will notice as you study the Metric System of measurement that the prefixes are the same for weight, length and volume. Only the units (meters, liters, grams) are changed for length and volume. The prefixes have the same meaning for *weight*, *length* or *volume*.



Weight: the amount of force on an object being pulled toward the center of the earth by gravity

Gram: the basic unit of measurement for weight in the metric system

Table 13.1

Prefix	Abbreviation	Meaning
kilo-	k	× 1000
hecto-	h	× 100
deka-	dk	× 10
unit (gram)		× 1
deci-	d	× $^1/_{10}$
centi-	c	× $^1/_{100}$
milli-	m	× $^1/_{1000}$

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

The first row of **Table 13.2** shows the abbreviations and units, in decreasing size, beginning with the largest and going to the smallest. The second row shows the meaning of each prefix abbreviation. The only difference between the tables in previous lessons and **Table 13.2** of this lesson is that the units are changed from *meters* (m) or *liters* (l) to *grams* (g).

Table 13.2

abbreviation:	**kg**	**hg**	**dkg**	**g**	**dg**	**cg**	**mg**
meaning:	× 1000	× 100	× 10	× 1	× $^1/_{10}$	× $^1/_{100}$	× $^1/_{1000}$
	⟵ Multiply				Divide ⟶		

Professor K points out that you must use the *Directional Method of Converting* to make conversions involving metric units of grams, meters or liters.

Example Set 1 shows you how to make conversions involving *grams*, the Metric System unit for weight.

Example Set 1

a) 4 kilograms = ??? grams
4 × 1000 = 4000 grams

In this problem the change from *kilograms* to *grams* is simple since kilo- means 1000 times. The *4* is multiplied by *1000*, to obtain *4000* grams as the answer.

4 kg × 10 = 40 hg
40 hg × 10 = 400 dkg
400 dkg × 10 = 4000 g

To change *kilograms* to *grams* using the *Directional Method of Converting*, the change requires moving from kg to g, or left to right (⟶), therefore, you need to multiply by *10* three times, since three prefix moves are needed to move from kg to g (kg $\xrightarrow{1}$ hg $\xrightarrow{2}$ dkg $\xrightarrow{3}$ g). Refer to **Table 13.2**.

b) 3400 dkg = ??? kg
3400 dkg ÷ 10 = 340 hg
340 hg ÷ 10 = 34 kg

In example b), you notice the move will be right to left from dkg to kg, which means division. Using the *Directional Method of Converting* you determine (by looking at **Table 13.2**) that two prefix moves are required to go from dkg to kg. (kg $\xleftarrow{2}$ hg $\xleftarrow{1}$ dkg). This gives an answer of *34* kg.

Example Set 2

a) 35 grams = ??? milligrams
35 × 1000 = 35000 milligrams

In this problem, the change from *grams* to *milligrams* is simple since *milli-* means $^1/_{1000}$. The *35* is multiplied by *1000*, to obtain *35,000* milligrams as the answer.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

35 g × 10 = 350 dg
350 dg × 10 = 3500 cg
3500 cg × 10 = 35000 mg

To change *grams* to *milligrams* using the *Directional Method of Converting*, the change is moving from g to mg, or left to right ($\longrightarrow$), therefore, you need to multiply by *10* three times, since three prefix moves are needed to move from g to mg (g $\xrightarrow{1}$ dg $\xrightarrow{2}$ cg $\xrightarrow{3}$ mg). Refer to **Table 13.2**.

b) 2500 dg = ??? dkg
2500 dg ÷ 10 = 250 g
250 g ÷ 10 = 25 dkg

In example b), you notice the move will be right to left from dg to dkg, which means division. Using the *Directional Method of Converting* you determine (by looking at **Table 13.2**) that two prefix moves are required to go from dg to dkg. (dkg $\xleftarrow{2}$ g $\xleftarrow{1}$ dg) This gives an answer of *25* dkg.

Work through the activities for this lesson to demonstrate your understanding of measurements of weight in the Metric System.

LIFE PRINCIPLE



"Making the right choices gives direction for your life."

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



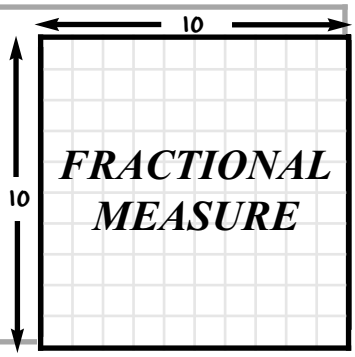
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Fractions

Lesson 14

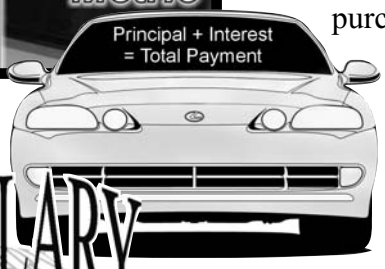
Chapter 3



Section 3

Lessons 11-15





Professor K realizes that everyday you encounter problems that involve measurements. Many of these problems also include fractions. Most cooks use recipes that require fractional portions of certain ingredients. When purchasing fresh food at the grocery store, the weight may be calculated in fractions of a pound. The odometer on Professor K's car measures not only *miles*, but also *tenths* of a mile. Professor K does not want to pay for fractional parts of a gallon of gasoline as if it were a whole gallon, so gas pumps measure out and compute the cost of his purchase in fractions of gallons. Often measurements are easier if you know the relationship certain fractions have to the measurement about which you are concerned. In this lesson Professor K will show you how to relate fractions to the measurements of length, volume, and weight in the English system of measurement.



Professor K

VOCABULARY

Devices: things constructed for particular purposes
qt: quart
pt: pint
T: tablespoon
tsp: teaspoon
yds: yards
Splice: to join together at the ends
California Gold Rush: a period of time in history of the United States, beginning in 1849 with the discovery of gold in California

VOLUME

In **Example Set 1**, Professor K will demonstrate that to get a clear fractional representation of a unit of volume measurement, you must convert the measurement to another unit.

Example Set 1

Professor K's wife was baking a large cake. The recipe required $\frac{3}{4}$ quart of milk, $1\frac{1}{2}$ cups of coconut oil, and $\frac{1}{2}$ teaspoon of salt. The only forms of measuring ***devices*** available for Professor K's wife to measure these ingredients were a cup and a teaspoon. The following example reveals mathematically how Professor K's wife would have to measure out the milk, oil, and salt precisely, so as not to guess.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Milk

1 ***qt*** = 2 ***pts***; 1 pt = 2 cups; therefore, 1 qt = 4 cups

$$\frac{1\text{ qt}}{1}\times\frac{2\text{ pts}}{1\text{ qt}}\times\frac{2\text{ cups}}{1\text{ pt}}=\frac{1\text{ qt}\times 2\text{ pts}\times 2\text{ cups}}{1\times 1\text{ qt}\times 1\text{ pt}}=\frac{2\times 2\text{ cups}}{1}=\frac{4\text{ cups}}{1}=4\text{ cups}$$

Notice how the conversions are set up so that the units (qt and pt), divide out (cancel) leaving only cups as the unit. Now Professor K's wife can find $^3/_4$ qt of milk by determining $^3/_4$ of *4* cups.

$$\frac{3}{4}\times\frac{4\text{ cups}}{1}=\frac{3\times 4\text{ cups}}{4\times 1}=\frac{3\text{ cups}}{1}=3\text{ cups}$$

Notice in this part of the problem that the *4*s divided out to become *1*. So Professor K's wife needs to add *3* cups of milk to the cake, because $^3/_4$ qt = 3 cups. The measurements of the other two ingredients, oil and salt, are found in a similar manner.



Oil

1 cup = 16 ***T***; 1 T = 3 ***tsp***; therefore, 1 cup = 48 tsp.

$$\frac{1\text{ cup}}{1}\times\frac{16\text{ T}}{1\text{ cup}}\times\frac{3\text{ tsp}}{1\text{ T}}=\frac{1\text{ cup}\times 16\text{ T}\times 3\text{ tsp}}{1\times 1\text{ cup}\times 1\text{ T}}=\frac{1\times 16\times 3\text{ tsp}}{1\times 1\times 1}=\frac{48\text{ tsp}}{1}=48\,\text{tsp}$$

Now Professor K's wife can find $^1/_2$ cups of oil by determining $^1/_2$ of 48 tsp.

$$\frac{1}{2}\times\frac{48\text{ tsp}}{1}=\frac{1\times 48\text{ tsp}}{2\times 1}=\frac{48\text{ tsp}}{2}=\frac{24\text{ tsp}}{1}=24\text{ tsp}$$

LENGTH

Professor K will demonstrate in **Example Set 2** that to get a clear fractional representation of a unit of length measurement, you must convert the measurement to another unit.

Example Set 2

Professor K's neighbor, Farmer Brown, wants to build a four-wire fence. The length of fence Farmer Brown wants to build is 700 yds. The barbed wire which Farmer Brown wants to use is available in rolls $^1/_4$ of a mile long. He can figure how many rolls of wire to be purchased in order to complete his new fence. (Farmer Brown can ***splice*** the ends of the wire together to make a strand of wire longer.) Remembering the conversion from Chapter 1, 1760 yds = 1 mile. Farmer Brown must calculate how many yards are in a roll of wire that is $^1/_4$ of a mile long.

$$\frac{1760\text{ yds}}{1}\times\frac{1}{4}=\frac{1760\text{ yds}\times 1}{1\times 4}=\frac{1760\text{ yds}}{4}=440\text{ yds}$$

The total amount of wire Farmer Brown will need for his fence must now be calculated. 700 yds (length of fence) × 4 (number of strands of wire) = 2800 yds total. The number of rolls of barbed wire needed can be calculated from the information above. *440* divides 2800 six times with a remainder of 160 (placed in fraction form).

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

$$\frac{2800\text{ yds}}{1}\times\frac{1\text{ roll}}{440\text{ yds}}=\frac{2800\text{ yds}\times 1\text{ roll}}{440\text{ yds}\times 1}=\frac{2800\text{ roll}}{440}=6\frac{160}{440}\text{ rolls}$$

Reduce the mixed number to simplest form.

$$6\frac{160}{440}\text{ rolls}=6\frac{4}{11}\text{ rolls of wire}$$

Because six complete and a partial roll of wire ($^{4}/_{11}$ of a roll) are needed, Farmer Brown will need to purchase 7 rolls of wire.

WEIGHT



In **Example Set 3**, Professor K will demonstrate that to get a clear fractional representation of a unit of weight measurement, you must convert the measurement to another unit. Follow the calculations as Professor K works through the example.

Example Set 3

In the days of the ***California Gold Rush***, a miner wanted to know the value of a large gold nugget weighing $2^{3}/_{4}$ lbs, if gold was selling for $35.00 per ounce. The first step to solving this problem was to change $2^{3}/_{4}$ lbs into an improper fraction. In the second step he had to convert $2^{3}/_{4}$ lbs into ounces. The final step will be to multiply the last answer obtained by $35.00. Follow the steps below to learn how the miner determined the value of his gold nugget.

$$2\frac{3}{4}\text{ lbs}=\frac{8}{4}+\frac{3}{4}\text{ lbs}=\frac{8+3}{4}\text{ lbs}=\frac{11}{4}\text{ lbs}$$

The calculation above changed $2^{3}/_{4}$ lbs into the improper fraction $^{11}/_{4}$ lbs. Then the miner used 1 lb = 16 ounces to calculate the nugget's weight in ounces

$$\frac{16\text{ oz}}{1\text{ lb}}\times\frac{11\text{ lbs}}{4}=\frac{11\text{ lbs}\times 16\text{ oz}}{1\text{ lb}\times 4}=\frac{11\times 4\text{ oz}}{1}=44\text{ oz}$$

Notice that *4* divides *16* four times and then *4* is multiplied by *11* to obtain *44* ounces of gold. For the final step you must multiply *44* oz times *$35.00* per ounce.

$$\frac{44\text{ oz}}{1}\times\frac{\$35}{1\text{ oz}}=\frac{44\text{ oz}\times\$35}{1\text{ oz}\times 1}=\frac{44\times\$35}{1}=\$1540\text{ worth of gold}$$

In the previous three examples notice how the units divide out. Units, like numbers, can be divided out (canceled) to become 1.

DECIMAL MEASUREMENTS - ENGLISH SYSTEM

Professor K has noticed that sometimes measurements in the English system are expressed in decimals. The most common use of decimals in the English system of measurement is in the calculation of how many miles a vehicle has traveled on a gallon of fuel. In **Example Set 4** Professor K shows such a calculation.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 4

Professor K visited his grandfather last weekend. Professor K traveled a total of 366 miles in the round trip to his grandfather's house and back home. Professor K filled his gasoline tank before he left and then refilled it immediately upon his return. Professor K's car used 20 gallons of gasoline for the trip. You may determine the miles per gallon the car consumed on Professor K's trip to visit his grandfather.

A *ratio* (or fraction) must be set up to show the relationship between *miles traveled* and *gallons of gasoline used*. This ratio is:

$$\frac{366 \text{ miles}}{20 \text{ gallons}} = 18.3 \text{ miles/gallon (read } \textit{eighteen and three tenths miles per gallon}\text{)}$$

The ratio is changed to a decimal by dividing *366* by *20* to obtain *18.3 miles/gallon*.

Demonstrate your understanding of fractions by working the activities for this lesson.

LIFE PRINCIPLE



"A good man is the sum of courage and kindness."

—Ronald Johnson

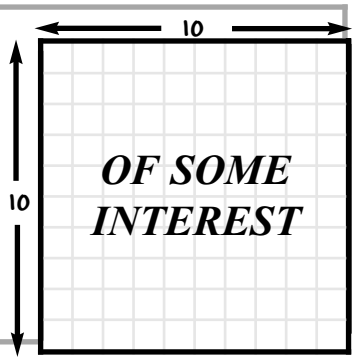
©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



Percent

Lesson 15

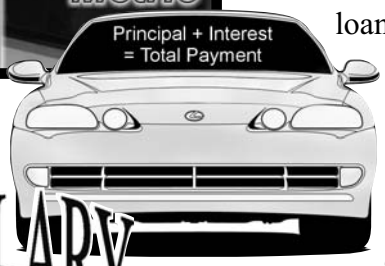
Chapter 3



Section 3

Lessons 11-15







Roberto needed to buy a new vehicle. One of the first things he wanted to do with new "wheels" was to take Rafael and his friends on a camping trip to the mountains. Roberto visited several car ***dealerships*** until he found just the right vehicle. Roberto then had a new problem. The dealership had a financing plan that required an interest rate of 10% on money loaned to new car buyers. Roberto believed he could get a better rate of ***interest*** at his bank. Roberto left the car dealer and drove across town to his bank, where he met his ***loan officer***. The loan officer informed Roberto that the bank would have to make a ***profit*** on the car ***loan*** to Roberto. The bank would charge an interest rate of 9%, compared to the 10% of the car dealer. The bank also allowed Roberto a longer period of time (the *term* of the loan) to pay back the money. Roberto had to make a decision. The longer term meant that his monthly payment would be smaller, but the extended payment term might require that he pay more total money for interest on the *principal* (the actual amount borrowed). The following examples show how Roberto made his decision on which loan to obtain.

VOCABULARY

Dealerships: places where vehicles are sold
Interest: extra money paid to banks for privilege of borrowing money
Loan officer: a person who arranges loans through a bank
Profit: amount remaining after all expenses have been paid
Loan: use of someone else's money to make a purchase
Simple interest: a flat fee charged by the bank regardless of the term

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Example Set 1

The ***simple interest*** charged on a loan is computed by multiplying the *principal* times the *rate* times the *time*: $I = Prt$.

a) P = \$25,000.00; r = 9%; t = 6

$I = Prt$

Step 1	I = (25000)(.09)(6)	Roberto figured the simple interest on the bank loan to make a comparison. The term of the bank loan was to be *6* years. Roberto placed the principal *$25,000.00*, rate, 9% =*.09* and the time, 6 years, in the formula for simple interest, $I = Prt$. The annual interest was \$2,250.00. (See Step 1.)
Step 2	I = (2250)(6) I = \$13,500.00	He then performed the multiplications to find the total interest was *$13,500.00* for the bank loan. (See Step 2.)

b) P = \$25,000.00; r = 10%; t = 4

$I = Prt$

Step 1	I = (25000)(.10)(4)	Roberto next calculated the interest for the loan from the car dealer. He placed the principal, *$25,000.00*, the rate, 10%, and the time, *4* years, in the formula, $I = Prt$. (See Step 1.)
Step 2	I = (2500)(4) I = \$10,000.00	He then performed the multiplications to find the total interest was *$10,000* for the loan from the car dealer. (See Step 2.)

In **Example Set 1**, Roberto was able to compare the total amount of interest for each loan. However, Roberto could not figure his monthly payment from this figure alone. In **Example Set 2**, Roberto used another formula to determine the entire amount of money that he would repay for each loan.

Example Set 2

The total amount repaid on a loan is calculated by using the formula: **amount repaid equals the original principal plus the total interest: $A = P + I = P + Prt$.**

a) $A = P + I$ A_b = \$25,000.00 + \$13,500.00 A_b = \$38,500.00	a) Roberto placed the original principal, *$25,000* and the interest figured in **Example Set 1** for the bank loan (*$13,500*) into the formula for calculating total amount repaid, A_b.

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

b) $A = P + I$
$A_c = \$25{,}000.00 + \$10{,}000.00$
$A_c = \$35{,}000$

b) Roberto placed the original principal, *$25,000* and the interest figured in **Example Set 1** for the loan from the car dealership, *$10,000*, into the formula for calculating total amount repaid, A_c.

In **Example Set 3**, Professor K will show you how Roberto estimated his monthly payment from the bank and the car dealership so he would know what he was facing and be able to stay within his budget. At the end of most months, Roberto has no more than $600 remaining in his checking account after paying all his bills.

Example Set 3

To calculate the monthly payment you must divide the total amount to be repaid ($A = P + I$) by the total number of months in the term of the loan.

a) $A_b = \$38{,}500$, $t = 72$ months

$$\frac{A_b}{t} = \frac{38500}{72} = \$534.72$$

a) Roberto placed the total amount to be repaid in **Example Set 2** for the bank loan of *$38,500*, into the formula for calculating monthly payment, A_b/t. He discovered that his monthly car payment would be *$534.72* for six years.

b) $A_c = \$35{,}000$, $t = 48$ months

$$\frac{A_c}{t} = \frac{35000}{48} = \$729.17$$

b) Roberto placed the total amount of *$35,000* to be repaid in **Example Set 2** for the loan from the car dealership into the formula for calculating monthly payment, A_c/t. He discovered that his payments would be *$729.17* for four years.

Show Professor K you understand simple interest problems by working the activities for this lesson.

LIFE PRINCIPLE



"Your interests are a reflection of who and what you are."

—Anonymous

©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.

Basic Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.

Mathematics Charts

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

©Copyright 2005, Paradigm Accelerated Curriculum

DUPLICATION BY WRITTEN PERMISSION ONLY.



©Copyright 2005, Paradigm Accelerated Curriculum DUPLICATION BY WRITTEN PERMISSION ONLY.